

LIPSCHITZ BASED POLYNOMIAL INTERPOLATION FOR STABLE INCOMPLETE DATA RECONSTRUCTION

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Abstrak. Rekonstruksi data tidak lengkap dapat mengalami fluktuasi berlebihan ketika observasi yang tersedia menjadi jarang atau tidak merata dan mengandung noise, sehingga interpolasi polinomial global menjadi sensitif terhadap variasi lokal. Penelitian ini mengusulkan pendekatan interpolasi polinomial berbasis Lipschitz, yang menggabungkan interpolasi Lagrange klasik dengan mekanisme pembatas gradien, yaitu membatasi perubahan lokal antar titik agar tidak melebihi batas Lipschitz yang ditentukan. Berbeda dari pendekatan rekonstruksi berbasis optimasi atau regularisasi iteratif, mekanisme yang diusulkan diterapkan sebagai proses stabilisasi sederhana setelah interpolasi tanpa memerlukan proses pelatihan atau optimasi berulang. Evaluasi dilakukan menggunakan data nonlinear sintetis dengan noise Gaussian pada tingkat kehilangan data sebesar 30%, 40%, dan 50%. Hasil menunjukkan adanya trade-off antara akurasi dan kehalusan rekonstruksi. Meskipun interpolasi klasik menghasilkan nilai RMSE dan MAE yang lebih rendah, metode berbasis Lipschitz menghasilkan nilai Total Variation (TV) yang lebih rendah pada sebagian besar skenario. Hasil ini menunjukkan bahwa pembatasan gradien efektif dalam mengendalikan fluktuasi lokal dan menghasilkan rekonstruksi yang lebih halus, dengan konsekuensi berupa penurunan akurasi titik. Dengan demikian, pendekatan yang diusulkan menyediakan mekanisme stabilisasi yang sederhana dan interpretatif untuk rekonstruksi data tidak lengkap dan noisy ketika pengendalian osilasi menjadi perhatian utama.

Kata Kunci: Rekonstruksi Data Tidak Lengkap, Kontinuitas Lipschitz, Data Noisy, Stabilitas Numerik, Interpolasi Polinomial

Abstract. Incomplete data reconstruction may experience excessive fluctuations when the available observations become sparse or irregular and are contaminated by noise, making global polynomial interpolation sensitive to local variations. This study proposes a Lipschitz-based polynomial interpolation approach that combines classical Lagrange interpolation with a gradient-limiting mechanism, which constrains local changes between consecutive points within a prescribed Lipschitz bound. Unlike reconstruction approaches based on optimization or iterative regularization, the proposed mechanism is applied as a simple post-interpolation stabilization process without requiring model training or repeated optimization. The method was evaluated using synthetic nonlinear data with Gaussian noise under missing-data rates of 30%, 40%, and 50%. The results reveal a trade-off between reconstruction accuracy and smoothness. While classical interpolation achieved lower RMSE and MAE values, the Lipschitz-based method produced lower Total Variation (TV) values in most scenarios. These results indicate that gradient limiting effectively controls local fluctuations and produces smoother reconstructions at the expense of reduced pointwise accuracy. Therefore, the proposed approach provides a simple and interpretable stabilization mechanism for incomplete and noisy data reconstruction when oscillation control is a primary concern.

Keywords: Incomplete Data Reconstruction; Lipschitz Continuity; Noisy Data; Numerical Stability; Polynomial Interpolation

A. Introduction

Incomplete data reconstruction is an important problem in signal reconstruction, particularly when an underlying signal must be recovered from partially observed and noisy data. Missing observations may arise from sensor malfunction, transmission disturbances, or limitations in data acquisition, potentially reducing reconstruction accuracy and introducing structural distortion or oscillatory artifacts (S. C. Wang et al., 2025; Y. Wang & Arora, 2024).



Incomplete measurements may also weaken structural consistency and increase reconstruction uncertainty (Göppel et al., 2022), while broader missing-data studies have highlighted their negative effects on reconstruction reliability (Hippert-Ferrer et al., 2026). Therefore, developing computationally efficient reconstruction methods that can recover missing data while controlling excessive local variations remains an important challenge.

Among various reconstruction approaches, interpolation-based methods remain widely used because of their simplicity, flexibility, and relatively low computational cost. Compared with optimization-based and learning-based reconstruction approaches, interpolation does not require large training datasets or iterative parameter optimization, making it suitable for controlled reconstruction problems with limited observations. In addition, its deterministic formulation provides a more transparent relationship between the available observations and the reconstructed values, allowing the effect of the proposed Lipschitz-based stabilization mechanism to be interpreted directly.

Polynomial interpolation is one of the most fundamental approximation techniques for reconstructing continuous functions from discrete observations. In numerical analysis, interpolation methods have been extensively utilized in approximation theory, optimization, stochastic integral equations, finite element analysis, surface reconstruction, forecasting, and function estimation (Boukhelkhal & Zeghdane, 2024; Li et al., 2024). Previous studies demonstrated that interpolation-based approximation methods can achieve accurate numerical solutions with relatively small absolute errors and good convergence behavior (Boukhelkhal & Zeghdane, 2024). Furthermore, interpolation error analysis and convergence behavior play important roles in determining the reliability and stability of numerical approximation methods. In image and spatial-data reconstruction, interpolation and sampling strategies are also frequently employed to estimate missing regions while preserving local continuity and structural consistency (Mariethoz & Renard, 2010).

Lagrange polynomial interpolation is particularly attractive because of its conceptual simplicity and ease of implementation for unequally spaced observations. Although spline and piecewise polynomial methods generally provide better local control and are more robust to local variations, Lagrange interpolation was selected in this study as a simple global baseline for examining the effect of the proposed Lipschitz-based stabilization mechanism. Its direct formulation allows changes introduced by the gradient-limiting procedure to be evaluated without additional local fitting or continuity constraints inherent in piecewise interpolation methods.

Previous studies have demonstrated that Lagrange interpolation can provide accurate approximations and favorable convergence behavior under appropriate interpolation settings (Boukhelkhal & Zeghdane, 2024). However, approximation accuracy and convergence are conceptually distinct from numerical stability, which concerns the sensitivity of the interpolant to perturbations in the observed data. The stability of polynomial interpolation is closely related to the conditioning of the interpolation process, where perturbations in the observed values may be amplified depending on the associated Lebesgue constant (Ibrahimoglu, 2016). Moreover, recent studies have specifically investigated additional procedures to reduce oscillatory behavior in Lagrange interpolation (de la Calle Ysern & Galán del Sastre, 2024). These considerations motivate the introduction of additional stabilization mechanisms when polynomial interpolation is applied to incomplete and noisy data.

Interpolation schemes based on carefully selected nodes, such as Chebyshev and Jacobi-Gauss nodes, are also frequently employed to reduce interpolation error and improve approximation stability. In forecasting and approximation problems, polynomial extrapolation models have also been used to generate smoother predictions and stable approximation trends under fluctuating observations (Zulfaa Sabina et al., 2025). In addition, comparative

implementation studies showed that Lagrange interpolation remains computationally efficient and practically applicable in numerical computation environments (Hidayat et al., 2025).

In practical missing-data problems, interpolation techniques such as linear interpolation, spline interpolation, nearest-neighbor interpolation, and polynomial fitting are frequently used because they preserve continuity and estimate unknown values efficiently from neighboring observations. Recent studies in photovoltaic power prediction emphasized that interpolation-based imputation methods are computationally efficient and effective for small missing intervals (Liu et al., 2025). However, their performance decreases significantly when the missing intervals become longer or when abrupt changes occur within the data distribution. Similar findings also appear in forecasting applications where approximation quality strongly depends on the smoothness and consistency of observed patterns (Bagaskara et al., 2024). In addition, recent deep-learning-based reconstruction studies showed that irregularly missing data remain difficult to recover accurately because sparse observations may weaken local structural information and continuity preservation (Chai et al., 2020).

Despite these advantages, classical polynomial interpolation remains sensitive to sparse observations and noisy data distributions. Under incomplete-data conditions, interpolation curves may exhibit excessive local oscillations, instability, and amplification of noise, especially when the number of available observations decreases significantly. Similar instability phenomena have also been observed in sparse-to-dense reconstruction problems, seismic interpolation, and inverse reconstruction frameworks, where insufficient sampling reduces reconstruction fidelity and generates oscillatory artifacts (Pan et al., 2025; Y. Wang et al., 2024). In global polynomial interpolation, Runge-type oscillations may appear near data boundaries and negatively affect reconstruction smoothness and approximation stability. It was further emphasized in (Li et al., 2024) that preserving smoothness and local structural consistency is essential for achieving reliable approximation results. Related reconstruction studies in inverse imaging also indicated that incomplete-data settings often require additional stability constraints to suppress artifacts and maintain feature consistency (Göppel et al., 2022; Kelly et al., 2017). These findings indicate that reconstruction quality depends not only on approximation accuracy but also on numerical smoothness and stability preservation.

To improve reconstruction robustness, several studies have introduced regularization and stability-oriented optimization strategies into interpolation and reconstruction frameworks. Sparse regularization methods are widely used to balance reconstruction fidelity and numerical stability in incomplete-data problems. Optimization techniques such as the Fast Iterative Shrinkage-Thresholding Algorithm (FISTA) have demonstrated effective convergence behavior and improved sparse reconstruction performance in seismic reconstruction studies (S. C. Wang et al., 2025). Other studies emphasized the importance of smooth low-variation mappings and continuity preservation in suppressing oscillatory artifacts and maintaining structural consistency in reconstructed signals (Li et al., 2024). In addition, machine-learning-based imputation methods have incorporated adversarial learning, gradient penalty mechanisms, and Lipschitz constraints to stabilize reconstruction processes and improve generated-data continuity under incomplete-data conditions (S. C. Wang et al., 2025). Recent surveys also highlighted that stability-preserving reconstruction remains one of the major challenges in modern missing-data recovery frameworks across signal processing and machine learning applications (Hippert-Ferrer et al., 2026).

One important concept frequently used in stability analysis is Lipschitz continuity, which restricts the maximum rate of change of a function. Lipschitz-related constraints are widely employed in optimization theory, inverse problems, regularization methods, and machine learning to improve convergence stability and control abrupt local variations. In reconstruction-related optimization models, Lipschitz conditions are commonly associated with gradient-control mechanisms and stable iterative updates (S. C. Wang et al., 2025). From a numerical-



analysis perspective, limiting excessive gradients can help preserve smoothness and reduce oscillatory behavior in reconstructed functions. Previous studies on Lipschitz interpolation also demonstrated that bounded Lipschitz mappings provide stable approximation behavior under stochastic perturbations and noisy observations (Huang et al., 2023). Moreover, stable interpolation operators should preserve smoothness while maintaining acceptable reconstruction fidelity (Li et al., 2024). Although Lipschitz continuity has been extensively applied in optimization and learning-based frameworks, its direct integration as a smoothness-control mechanism for classical polynomial interpolation in incomplete-data reconstruction remains relatively underexplored.

Previous studies have applied Lipschitz-related constraints to control gradients and improve stability in optimization and reconstruction frameworks (S. C. Wang et al., 2025; Huang et al., 2023), while other studies have introduced additional procedures to reduce oscillations in Lagrange interpolation (de la Calle Ysern & Galán del Sastre, 2024). However, the use of a Lipschitz-inspired gradient constraint as a post-interpolation stabilization mechanism for global polynomial reconstruction under incomplete and noisy observations remains unexplored. Therefore, this study introduces a Lipschitz-based polynomial interpolation framework that applies gradient limiting to suppress excessive local variations and evaluates the resulting trade-off between reconstruction accuracy (RMSE and MAE) and smoothness (TV).

B. METHODS

This study employed a quantitative computational approach to investigate the stability of polynomial interpolation for incomplete data reconstruction under noisy conditions. The proposed method combines classical polynomial interpolation with a Lipschitz-based smoothness constraint to reduce excessive local oscillation during reconstruction. The study focused on one-dimensional synthetic continuous data because synthetic functions allow controlled evaluation of reconstruction accuracy, smoothness behavior, and numerical stability under varying missing-data scenarios.

The original signal was generated from a continuous nonlinear function defined on the interval $[0,1]$. Let the original dataset be represented by

$$D = \{(x_i, y_i)\}_{i=1}^N$$

where x_i denotes the sampling points and $y_i = f(x_i)$ represents the observed signal values. In this study, the continuous signal was generated using the function

$$f(x) = \sin(2\pi x) + 0.3 \sin(6\pi x)$$

which contains both low-frequency and high-frequency components. This characteristic allows the experiment to simulate realistic reconstruction situations involving smooth global trends together with local fluctuations. The signal was discretized into uniformly spaced observations using

$$x_i = \frac{i}{N-1}, \quad i = 0, 1, 2, \dots, N-1$$

with N denoting the total number of sampled points.

To simulate incomplete-data conditions, subset observations was randomly removed from the original dataset. Let

$$\Omega \subset \{1, 2, \dots, N\}$$

denote the index set of available observations. The missing-data was controlled by randomly removing a proportion of r of the total observations, where $r \in \{0.3, 0.4, 0.5\}$. The observed incomplete dataset can therefore be written as

$$D_{obs} = \{(x_i, y_i) : i \in \Omega\}$$



To further evaluate robustness under noisy conditions, additive Gaussian noise was incorporated into the observed signal. The noisy observations were defined as

$$\tilde{y}_i = y_i + \epsilon_i$$

where

$$\epsilon_i \sim \mathcal{N}(0, \sigma^2)$$

and σ represents the noise level. The inclusion of noise allows the proposed method to be evaluated under more realistic numerical conditions where observed data are contaminated by perturbation and measurement uncertainty.

The first reconstruction approach employed in this study was classical polynomial interpolation. Suppose that $n + 1$ observed data points are available. The interpolation polynomial $P_n(x)$ of degree at most n is defined using the Lagrange formulation

$$P_n(x) = \sum_{i=0}^n y_i L_i(x)$$

where

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^n \frac{x - x_j}{x_i - x_j}$$

denotes the i -th Lagrange basis polynomial. Classical polynomial interpolation was selected as the baseline method because it provides exact fitting at observed points and is widely used in approximation theory and numerical reconstruction problems.

However, classical interpolation may generate excessive oscillation when the observed data become sparse or noisy. To improve reconstruction smoothness, this study introduced a Lipschitz-based stabilization mechanism. A function $g(x)$ is said to satisfy the Lipschitz condition if there exists a constant $K > 0$ such that

$$|g(x) - g(y)| \leq K|x - y|$$

for all x, y in the domain. The Lipschitz constant K controls the maximum allowable rate of change of the reconstructed function. Smaller values of K enforce smoother reconstructions, while larger values allow greater local variation.

To incorporate this principle into interpolation reconstruction, the local gradient of the interpolated signal was constrained during post-processing. Let the reconstructed polynomial values be denoted by $P_n(x_i)$. The local discrete gradient was approximated using

$$\Delta_i = P_n(x_{i+1}) - P_n(x_i)$$

the Lipschitz-based reconstruction then modifies the local variation according to

$$\hat{\Delta}_i = \text{clip}(\Delta_i, -K\Delta x, K\Delta x)$$

where $\Delta x = x_{i+1} - x_i$ and $\text{clip}(\cdot)$ restrict the gradient magnitude to the interval $[-K\Delta x, K\Delta x]$.

The stabilized reconstructed signal was subsequently computed recursively by

$$\hat{y}_{i+1} = \hat{y}_i + \hat{\Delta}_i$$

this procedure suppresses excessive local oscillations while preserving the global trend of the reconstructed signal.

The reconstruction performance was quantitatively evaluated using Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and Total Variation (TV). RMSE measures the overall reconstruction error magnitude and is defined as

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2}$$

meanwhile, MAE measures the average absolute reconstruction error:



$$MAE = \frac{1}{N} \sum_{i=1}^N |y_i - \hat{y}_i|$$

to evaluate reconstruction smoothness and oscillatory behavior, Total Variation was computed using

$$TV = \sum_{i=1}^{N-1} |\hat{y}_{i+1} - \hat{y}_i|$$

lower TV values indicate smoother reconstruction results with reduced local oscillation.

All numerical experiments were implemented in Python using NumPy, SciPy, and Matplotlib libraries. Reconstruction experiments were conducted under several missing-data rates and noisy conditions using fixed random seeds to ensure reproducibility and consistent comparison between classical polynomial interpolation and the proposed Lipschitz-based interpolation method.

C. Results And Discussion

1. Experimental Setup

This study evaluates the performance of the proposed Lipschitz-based polynomial interpolation method under incomplete and noisy data conditions. Synthetic data are generated using a smooth nonlinear function with additive Gaussian noise of intensity 0.1. To simulate data incompleteness, three missing data scenarios are considered, namely 30%, 40%, and 50% missing rates.

For reproducibility, all experiments are conducted using a fixed random seed (seed = 100). The reconstruction performance is evaluated using three metrics: Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and Total Variation (TV). RMSE and MAE measure reconstruction accuracy, while TV is used to quantify the smoothness of the reconstructed function.

2. Quantitative Results

Table 1 presents the reconstruction performance of classical polynomial interpolation and the proposed Lipschitz-based method under different missing data scenarios.

Table 1. Reconstruction Accuracy and Smoothness

Missing Rate	Noise	RMSE		MAE		TV	
		Poly	Lipschitz	Poly	Lipschitz	Poly	Lipschitz
30%	0.1	0.160	0.171	0.133	0.138	6.893	6.852
40%	0.1	0.168	0.195	0.135	0.154	6.953	6.950
50%	0.1	0.175	0.206	0.141	0.164	7.355	7.291

From Table 1, it can be observed that classical polynomial interpolation consistently achieves lower RMSE and MAE values across all scenarios. This indicates that polynomial interpolation provides better pointwise fitting accuracy compared to the Lipschitz-based method.

However, a different trend is observed in terms of total variation (TV). The Lipschitz-based interpolation generally produces lower TV values, particularly in the 30% and 50% missing data scenarios. This suggests that the proposed method is effective in reducing local oscillations and producing smoother reconstructions.

At the 40% missing rate, the difference in TV between the two methods is relatively small, indicating that the smoothing effect of the Lipschitz constraint is less pronounced under moderate data loss.



3. Visual Analysis

Figure 1-Figure 3 illustrate the reconstruction results for different missing data scenarios.

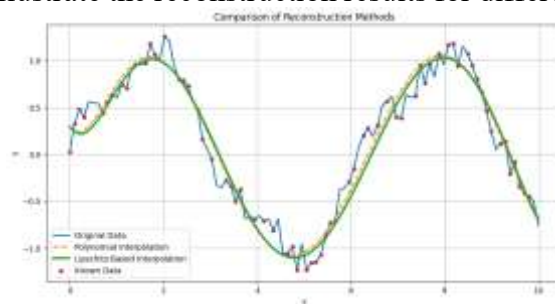


Figure 1. Reconstruction Results for 30% Missing Data

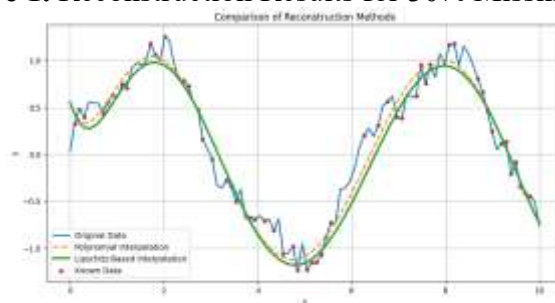


Figure 2. Reconstruction Results for 40% Missing Data

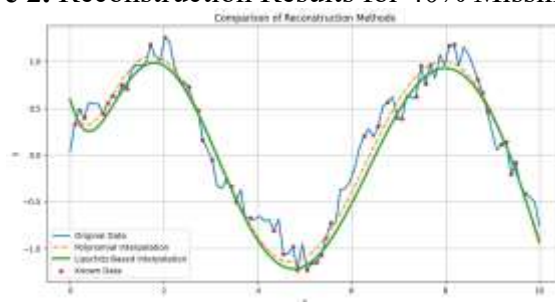


Figure 3. Reconstruction Results for 50% Missing Data

From the visual comparison, classical polynomial interpolation tends to exhibit noticeable oscillatory behavior, especially in regions where data points are sparse. These oscillations become more apparent as the missing rate increases.

In contrast, the Lipschitz-based method produces smoother curves with reduced local fluctuations. The reconstructed curves follow the general trend of the original data while avoiding excessive oscillation. This behavior is consistent with the imposed Lipschitz constraint, which limits the rate of change of the interpolating function.

However, the smoothing effect also leads to slight deviations from the original data, particularly near peaks and valleys. This explains the slightly higher RMSE and MAE values observed in the Lipschitz-based method.

4. Discussion

The experimental results reveal a clear trade-off between accuracy and smoothness in interpolation-based reconstruction. At the 30% missing rate, classical polynomial interpolation achieved lower RMSE (0.160 vs. 0.171) and MAE (0.133 vs. 0.138), whereas the Lipschitz-based method produced a lower TV value (6.852 vs. 6.893). A similar pattern was observed at the 40% missing rate, where classical interpolation maintained better accuracy with RMSE and MAE values of 0.168 and 0.135, compared with 0.195 and 0.154 for the Lipschitz-based method, while the TV values were nearly identical (6.953 vs. 6.950). At the 50% missing rate, the accuracy difference remained evident, with lower RMSE (0.175 vs. 0.206) and MAE (0.141 vs. 0.164) for classical interpolation, whereas the Lipschitz-based method again achieved a

lower TV value (7.291 vs. 7.355). These results demonstrate that the improved smoothness obtained through gradient limiting is accompanied by a reduction in pointwise reconstruction accuracy.

However, this flexibility may also increase sensitivity to sparse and irregularly distributed observations, particularly when noise perturbs the available data. As the missing rate increases, the reduced number and uneven distribution of interpolation points can make the global polynomial more susceptible to local fluctuations and Runge-type oscillatory behavior. This tendency is visible in Figures 1–3, where stronger fluctuations appear in regions with fewer observed points, and is also reflected numerically by the increasing TV values of classical interpolation from 6.893 at 30% missing data to 6.953 and 7.355 at 40% and 50%, respectively.

In contrast, the proposed Lipschitz-based interpolation controls local fluctuations through a post-interpolation gradient-limiting mechanism. After the Lagrange polynomial produces the initial reconstructed values, the local rate of change between consecutive points is computed and constrained within a prescribed Lipschitz bound. When the magnitude of a local gradient exceeds this bound, it is truncated before being used to reconstruct the subsequent value. Consequently, abrupt changes generated by the global polynomial are prevented from propagating across neighboring points, thereby reducing excessive local oscillations and producing smoother reconstructions, as reflected by the lower total variation values in most cases.

This smoothing effect, however, comes at the cost of reduced fitting flexibility, leading to higher reconstruction errors. The results suggest that the choice between the two approaches should depend on the characteristics and objectives of the data reconstruction problem. Classical polynomial interpolation may be preferable when pointwise accuracy and the preservation of local variations are the primary objectives. In contrast, the Lipschitz-based approach may be more suitable for incomplete and noisy data where excessive oscillations and smoothness preservation are of greater concern, particularly as data sparsity increases. Thus, the proposed method provides a practical stabilization mechanism when controlling local fluctuations is considered more important than achieving the lowest pointwise reconstruction error.

D. Conclusion

This study proposed a Lipschitz-based polynomial interpolation method for reconstructing incomplete and noisy data. The experimental results showed that the proposed method was able to reduce local oscillations and produce smoother reconstruction results, as indicated by lower total variation values in most scenarios. Although classical polynomial interpolation achieved better RMSE and MAE values, the Lipschitz-based approach provided improved smoothness and numerical stability. These findings indicate that the proposed method can serve as a simple stabilization strategy for polynomial interpolation under incomplete data conditions. Future studies may investigate adaptive Lipschitz control and comparisons with other interpolation methods.

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