Proximal: Jurnal Penelitian Matematika dan Pendidikan Matematika
ISSN 26158132 (cetak)
ISSN 26157667 (online)

AN SEIR MODEL FOR MEASLES TRANSMISSION IN NORTH SUMATRA USING THE FOURTH-ORDER RUNGE–KUTTA METHOD FOR AN OPTIMAL CONTROL STRATEGY

Dwi Andini[1], R. Maisaroh Rezyekiyah Siregar[2]
Department of Mathematics Universitas Islam Negeri Sumatera Utara[1,2]
Email: andinidwi097@gmail.com[1], maisarohsrg@uinsu.ac.id[2]

Corresponding Author: Dwi Andini **email:** andinidwi097@gmail.com

Abstract. Measles remains one of the major public health concerns in North Sumatra Province. This study aims to analyze the dynamics of measles transmission using the Susceptible–Exposed–Infected–Recovered (SEIR) model, which is solved numerically using the Fourth-Order Runge–Kutta (RK4) method and incorporates a control variable to evaluate the effect of reducing social contact on disease transmission. The study employs secondary data on measles cases obtained from the North Sumatra Provincial Health Office. Numerical simulations were performed using MATLAB with a step size of h = 1 day over a simulation period of 100 days. The results show that the SEIR model is capable of describing the dynamics of measles transmission through changes in the number of individuals in each compartment. The RK4 method provides stable numerical solutions, while the implementation of the control variable effectively reduces the number of individuals in the Exposed (E) and Infected (I) compartments. Among the four control scenarios examined, namely u = 0, u = 0.3, u = 0.6, and u = 0.9, the control level of u = 0.9 was found to be the most effective in suppressing the spread of measles.

Keywords: **Measles, SEIR model, Fourth-Order Runge–Kutta method, Control variable, Numerical simulation.**

A. Introduction

Infectious diseases remain one of the major public health concerns that require serious attention in many countries, including Indonesia. These diseases can spread rapidly through direct or indirect contact between individuals, potentially leading to outbreaks within a population (Beni et al., 2021). One of the infectious diseases that continues to occur is measles. Measles is a viral infectious disease caused by the measles virus. The virus is transmitted through direct contact as well as through respiratory droplets generated by coughing and sneezing. It is susceptible to acidic substances, chemical agents, ultraviolet radiation, and heat (Nisa et al., 2022). Measles is an infectious disease caused by the measles virus, a member of the *Paramyxoviridae* family, and is characterized by a high transmission rate. The disease spreads through respiratory droplets released when an infected individual coughs, sneezes, or talks. Measles primarily affects individuals who lack immunity to the virus, either because they have not received vaccination or have not acquired natural immunity (Asy-syifaa et al., 2024).

The high transmissibility of measles makes it capable of causing outbreaks if effective control measures are not implemented. Moreover, measles can lead to severe complications, including pneumonia, diarrhea, respiratory tract infections, encephalitis, and even death (Xerri et al., 2020). Therefore, measles remains a significant public health concern at both the global and national levels.

In Indonesia, measles continues to pose a public health challenge, including in North Sumatra Province, where the spread of infectious diseases is facilitated by high population density, population mobility, and uneven vaccination coverage (Pasaribu & Husein, 2024). Consequently, analyzing the transmission dynamics of measles is essential for developing effective prevention and control strategies (Zahrah et al., 2023).One of the approaches used to analyze the transmission dynamics of measles is mathematical modeling. In epidemiology, mathematical models are employed to describe disease transmission by representing changes in population compartments over time (Costa et al., 2021). To reduce the increase in measles cases, a control variable representing social contact reduction—such as maintaining physical distance and avoiding crowded places—is incorporated into the model, allowing the effectiveness of control strategies to be evaluated quantitatively.

One of the most widely used epidemiological models for analyzing infectious disease transmission is the Susceptible–Exposed–Infected–Recovered (SEIR) model. The SEIR model divides the population into four compartments: susceptible individuals, exposed individuals in the incubation period, infected individuals, and recovered individuals (Chebotaeva et al., 2025). This model is appropriate for measles because the disease has an incubation period before infected individuals develop symptoms and become capable of transmitting the virus to others. The interactions among the compartments in the SEIR model are represented by a system of nonlinear differential equations, which are solved numerically using the Fourth-Order Runge–Kutta (RK4) method due to its high level of accuracy (Vitanov, 2023). This study integrates the SEIR model with a control variable representing social contact reduction to analyze the transmission dynamics of measles in North Sumatra Province. The model is solved numerically using the Fourth-Order Runge–Kutta (RK4) method implemented in MATLAB based on secondary data obtained from the North Sumatra Provincial Health Office. The findings of this study are expected to contribute to the development of epidemiological modeling and to support the formulation of effective measles prevention and control strategies.

B. Research Method

This study employs the Susceptible–Exposed–Infected–Recovered (SEIR) model to analyze the transmission of measles in North Sumatra Province using secondary data obtained from the North Sumatra Provincial Health Office. The system of differential equations is solved numerically using the Fourth-Order Runge–Kutta (RK4) method implemented in MATLAB. In addition, a control variable is incorporated to evaluate the effect of reducing social contact on disease transmission. The initial conditions of the model are established based on measles case data for the year 2025. The research procedures are as follows:

a. Conduct a literature review on the epidemiology of measles, the SEIR model, and the Fourth-Order Runge–Kutta (RK4) method.
b. Collect secondary data on measles cases from the North Sumatra Provincial Health Office and determine the initial conditions and model parameters.
c. Develop the SEIR mathematical model based on epidemiological assumptions.
d. Determine the model parameters and solve the system of differential equations numerically using the Fourth-Order Runge–Kutta (RK4) method implemented in MATLAB.
e. Perform numerical simulations to analyze the dynamics of measles transmission and present the results in graphical form.
f. Incorporate a control variable to evaluate the effect of reducing social contact on the transmission of measles.
g. Interpret the simulation results and draw conclusions regarding the dynamics of measles transmission in North Sumatra Province.

C. Results and Analysis

The measles transmission model employed in this study is the Susceptible–Exposed–Infected–Recovered (SEIR) model with the inclusion of a control variable (u) representing social contact reduction measures. The model is used to describe the transition of individuals from the susceptible compartment to the recovered compartment. The formulation of the model is based on the following assumptions:

a. The total population is assumed to remain constant, such that $(\boldsymbol{N} = \boldsymbol{S} + \boldsymbol{E} + \boldsymbol{I} + \boldsymbol{R})$.
b. The birth rate and the natural death rate are assumed to be equal and are represented by the parameter $(\boldsymbol{\mu})$
c. Susceptible individuals (S) become exposed after coming into contact with infected individuals (I).
d. Exposed individuals (E) undergo an incubation period before becoming infectious.
e. Individuals in the Exposed (E) compartment move to the Infected (I) compartment at a rate of $(\boldsymbol{\varepsilon})$.
f. Infected individuals (I) recover and move to the Recovered (R) compartment at a recovery rate of $(\boldsymbol{\gamma})$.
g. The control variable $(\boldsymbol{u})$represents the level of social contact reduction, such as maintaining physical distance, avoiding crowded places, and limiting interpersonal interactions, thereby reducing the disease transmission rate.
h. Recovered individuals (R) are assumed to acquire immunity and do not return to the Susceptible (S) compartment during the observation period.
i. All model parameters are assumed to be positive.

Based on these assumptions, the SEIR model used in this study is formulated as follows:

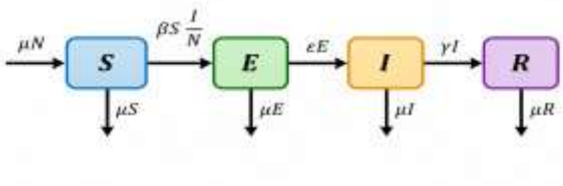

Figure 1. Measles Transmission Diagram

Mathematically the SEIR model is formulated by taking into account the birth rate and natural death rate. To support the implementation of an optimal control strategy, a control variable (u) is incorporated into the model to represent social contact reduction measures, such as maintaining physical distancing and avoiding crowded places. This control variable aims to reduce the transmission rate of measles and is expressed as follows:

$$\frac{dS}{dt} = \mu N - (1-u)\frac{\beta SI}{N} - \mu S \tag{1}$$

$$\frac{dE}{dt} = (1-u)\frac{\beta SI}{N} - (\mu + \varepsilon)E \tag{2}$$

$$\frac{dI}{dt} = \varepsilon E - (\mu + \gamma)I \tag{3}$$

$$\frac{dR}{dt} = \gamma I - \mu R \tag{4}$$

where:
S(t) = number of susceptible individuals (Susceptible);
E(t) = number of exposed individuals (Exposed);
I(t) = number of infected individuals (Infected);
R(t) = number of recovered individuals (Recovered);
β= disease transmission rate;
ε= rate at which exposed individuals become infected;
γ = recovery rate;
μ = birth and natural death rate;
$u(t)$= control variable.

https://doi.org/10.30605/proximal.v9i2.9552

The consistency of the model is verified by summing the four differential equations as follows:

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dR}{dt}$$

Substituting Equations (1)–(4) yields:

$$= \left[\mu N - (1-u)\left(\frac{\beta SI}{N}\right) - \mu S\right] + \left[(1-u)\left(\frac{\beta SI}{N}\right) - (\mu + \varepsilon)E\right] + [\varepsilon E - (\mu + \gamma)I] + [\gamma I - \mu R]$$

$$= \mu N - \mu S - \mu E - \mu I - \mu R$$

$$= \mu N - \mu(S + E + I + R)$$

$$= \mu N - \mu N = 0$$

The result indicates that the total population remains constant throughout the simulation period. Therefore, the model satisfies the assumption of a closed population, in which changes in the number of individuals occur solely due to transitions between compartments, while births and natural deaths remain balanced through the parameter μ.

Initial Conditions and Model Parameter Determination

The initial conditions were established based on the 2025 measles case data:

$S(0) = N - E(0) - I(0) - R(0)$

$= 15.785.839 - 1.676 - 838 - 0$

$= 15.783.325$

$E(0)=2\times I(0)=2\times 838 = 1.676$

(The initial exposed population was assumed to be twice the initial infected population as the baseline assumption because the number of exposed individuals was not directly available from the reported data.)

I(0)=838 (active infected cases based on the 2025 measles case data the North Sumatra Provincial Health Office)

R(0) = 0 (assumed as the initial condition of the model because data on the number of recovered individuals at the beginning of the simulation period were not reported by the North Sumatra Provincial Health Office) (Lim et al., 2025).

Table 1. SEIR Model Parameter Values

Parameter	Value	Description
β	0,4	Measles transmission rate
ε	0,2	Progression rate from $E \to I$
γ	0,1	Recovery rate
μ	2.7397×10^{-5}	Birth and natural death rate
$u(t)$	0 – 1	Social contact control variable
N	15.785.839	Total population of North Sumatra
h	1	RK4 step size
T	100	Simulation period (days)

.Based on the specified initial conditions and model parameters, the SEIR system of differential equations was solved numerically using the Fourth-Order Runge–Kutta (RK4) method with a step size of h=1 day over a simulation period of 100 days. The natural birth and death rate was specified as $\mu = 0.01$ per year and converted into a daily rate to ensure consistency with the time unit used in the model. Thus, the daily natural birth and death rate was set to $\mu = 0.01/365 = 2.7397 \times 10^{-5}$ day^{-1}. The initial simulation was performed under the uncontrolled scenario ($u = 0$). Subsequently, the initial conditions and model parameters were substituted into the system of differential equations as the basis for the numerical computations using the RK4 method.

Numerical Solution Using the Fourth-Order Runge–Kutta Method

After the SEIR model, initial conditions, and model parameters had been established, the system of differential equations was solved numerically using the Fourth-Order Runge–Kutta (RK4) method. The simulation was performed with a step size of $h = 1$ day over a period of 100 days to obtain the dynamics of measles transmission. The natural birth and death rate was converted from 0.01 per year to 2.7397×10^{-5}per day to maintain consistency with the daily time scale of the simulation.

RK4 Formulation for the SEIR System

The system of ordinary differential equations (ODE) in Equations (4.1)–(4.4) can be written in vector form as follows:

$$\frac{dy}{dt} = f(t, y), y(0) = y^0 \tag{5}$$

Where $y = \begin{bmatrix} S \\ E \\ I \\ R \end{bmatrix}, y^0 = \begin{bmatrix} 15.783.325 \\ 1.676 \\ 838 \\ 0 \end{bmatrix}$ dan $f = \begin{bmatrix} f^1 \\ f^2 \\ f^3 \\ f^4 \end{bmatrix}$

where:

$$f^1(t, y) = \mu N - (1-u)\frac{\beta \cdot S \cdot I}{N} - \mu S$$
$$f^2(t, y) = (1-u)\frac{\beta \cdot S \cdot I}{N} - (\mu + \varepsilon)E$$
$$f^3(t, y) = \varepsilon E - (\mu + \gamma)I$$
$$f^4(t, y) = \gamma I - \mu R$$

For each time step from $t_n ke t_n{}^{+1} = t_n + h$, the RK4 method computes:

$$k^1 = f(t_n, y_n) \tag{6}$$
$$k^2 = f\left(t_n + \frac{h}{2}, y_n + \left(\frac{h}{2}\right)k^1\right) \tag{7}$$
$$k^3 = f\left(t_n + \frac{h}{2}, y_n + \left(\frac{h}{2}\right)k^2\right) \tag{8}$$
$$k^4 = f(t_n + h, y_n + h \cdot k^3) \tag{9}$$
$$y_n{}^{+1} = y_n + \left(\frac{h}{6}\right)(k^1 + 2k^2 + 2k^3 + k^4) \tag{10}$$

Each k_i is a 4×1 vector containing the slope values for each compartment.

First Iteration of the Fourth-Order Runge–Kutta Method

The first iteration was performed over the time interval from $t_0 = 0$ to $t_1 = 1$ day. Based on the initial conditions and the parameter values specified in Table 4.2, the following values were used:

$$S_0 = 15.783.325,$$
$$E_0 = 1.676,$$
$$I_0 = 838,$$

$$R_0 = 0,$$

with the following parameter values:

$N = 15.785.839$, $\beta = 0{,}4$, $\varepsilon = 0{,}2$, $\gamma = 0{,}1$, $\mu = 2.7397 \times 10^{-5}$, $h = 1$,

together with the control variable value:

$$u = 0$$

which represents the uncontrolled scenario. Subsequently, the four slope vectors are computed as follows (k_1, k_2, k_3, dan k_4) for each compartment using the Fourth-Order Runge–Kutta (RK4) method.

Step 1. Computing k_1

The value of k_1 is obtained by evaluating the model function at the initial conditions, namely:

$$k_{1S} = f_1(S_0, E_0, I_0, R_0)$$
$$k_{1E} = f_2(S_0, E_0, I_0, R_0)$$
$$k_{1I} = f_3(S_0, E_0, I_0, R_0)$$
$$k_{1R} = f_4(S_0, E_0, I_0, R_0)$$

Substituting the parameter values into the model equations yields:

$$\begin{aligned} k_{1S} &= 2.7397 \times 10^{-5}\,(15.785.839) - \frac{0{,}4(15.783.325)(838)}{15.785.839} \\ &\quad - 2.7397 \times 10^{-5}\,(15.783.325) \\ &= 432{,}49 - 335{,}15 - 432{,}42 \\ &= -335{,}08 \end{aligned}$$

$$k_{1E} = \frac{0{,}4(15.783.325)(838)}{15.785.839} - 0{,}200027397(1.676)$$
$$= 335{,}15 - (0{,}200027397)(1.676)$$
$$= 335{,}15 - 335{,}25$$
$$= -0{,}10$$

$$k_{1I} = 0{,}2(1.676) - 2.7397 \times 10^{-5} + 0{,}1(838)$$
$$= 335{,}20 - 83{,}82$$
$$= 251{,}38$$

$$k_{1R} = 0{,}1(838) - 2.7397 \times 10^{-5}(0)$$
$$= 83{,}80$$

Thus, it is obtained that:

$$k_1 = \begin{bmatrix} -335{,}08 \\ -0{,}10 \\ 251{,}38 \\ 83{,}80 \end{bmatrix}$$

Step 2. Computing k_2

The value of k_2 is computed using the midpoint values, namely:

$$k_2 = f\left(t_0 + \frac{h}{2}, y_0 + \frac{h}{2} k_1\right)$$

Before computing k_2, the midpoint values for each compartment are first determined as follows:

$$S = S_0 + \frac{h}{2}k_{1S} = 15.783.325 + \frac{1}{2}(-335{,}08) = 15.783.157{,}46$$
$$E = E_0 + \frac{h}{2}k_{1E} = 1.676 + \frac{1}{2}(-0{,}10) = 1.675{,}95$$
$$I = I_0 + \frac{h}{2}k_{1I} = 838 + \frac{1}{2}(251{,}38) = 963{,}69$$
$$R = R_0 + \frac{h}{2}k_{1R} = 0 + \frac{1}{2}(83{,}80) = 41{,}90$$

Next, these values are substituted into the model equations.

$$k_{2S} = 2.7397 \times 10^{-5}(15.785.839) - \frac{0{,}4(15.783.157{,}46)(963{,}69)}{15.785.839} - 2.7397 \times 10^{-5}(15.785.839)$$

$$= 432{,}49 - 385{,}34 - 432{,}42$$
$$= -385{,}34$$

$$k_{2E} = \frac{0{,}4(15.783.157{,}46)(963{,}69)}{15.785.839} - 0{,}200027397(1.675{,}95)$$
$$= 385{,}34 - 335{,}17$$
$$= 50{,}17$$

$$k_{2I} = 0{,}2(1.675{,}95) - (0{,}100027397)(963{,}69)$$
$$= 335{,}19 - 96{,}40$$
$$= 238{,}79$$

$$k_{2R} = 0{,}1(963{,}69) - (2{,}7397 \times 10^{-5})(41{,}90)$$
$$= 96{,}37 - 0{,}0011$$
$$= 96{,}37$$

Thus, it is obtained that:

$$k_2 = \begin{bmatrix} -385{,}34 \\ 50{,}17 \\ 238{,}79 \\ 96{,}37 \end{bmatrix}$$

Step 3. Computing $\mathbf{k_3}$

The value of k_3 is computed using the midpoint values obtained from k_2, namely:

$$k_3 = \begin{bmatrix} -382{,}82 \\ 42{,}63 \\ 244{,}45 \\ 95{,}74 \end{bmatrix}$$

Step 4. Computing $\mathbf{k_4}$

The value of k_4 is computed using Equation (4.9) with the aid of MATLAB.

$$k_4 = \begin{bmatrix} -432{,}82 \\ 89{,}13 \\ 235{,}45 \\ 108{,}24 \end{bmatrix}$$

Step 5. Computing the First Iteration Solution

After obtaining the values of k_1, k_2, k_3, dan k_4, The first iteration solution is then computed using the Fourth-Order Runge–Kutta (RK4) equation as follows:

$$y_1 = y_0 + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = \begin{bmatrix} -335{,}08 \\ -0{,}10 \\ 251{,}38 \\ 83{,}80 \end{bmatrix}, 2k_2 = \begin{bmatrix} -770{,}67 \\ 100{,}35 \\ 477{,}59 \\ 192{,}74 \end{bmatrix}, 2k_3 = \begin{bmatrix} -765{,}64 \\ 85{,}26 \\ 488{,}90 \\ 191{,}48 \end{bmatrix} k_4 = \begin{bmatrix} -432{,}82 \\ 89{,}13 \\ 235{,}45 \\ 108{,}24 \end{bmatrix}$$

$k_1 + 2k_2 + 2k_3 + k_4 =$

$$\begin{bmatrix} -335{,}08 \\ -0{,}10 \\ 251{,}38 \\ 83{,}80 \end{bmatrix} + 2\begin{bmatrix} -770{,}67 \\ 100{,}35 \\ 477{,}59 \\ 192{,}74 \end{bmatrix} + 2\begin{bmatrix} -765{,}64 \\ 85{,}26 \\ 488{,}90 \\ 191{,}48 \end{bmatrix} + \begin{bmatrix} -432{,}82 \\ 89{,}13 \\ 235{,}45 \\ 108{,}24 \end{bmatrix}$$

Thus, the following result is obtained: $= \begin{bmatrix} -2304{,}21 \\ 274{,}64 \\ 1453{,}32 \\ 576{,}25 \end{bmatrix}$

Then, $\frac{1}{6}\begin{bmatrix} -2304{,}21 \\ 274{,}64 \\ 1453{,}32 \\ 576{,}25 \end{bmatrix} = \begin{bmatrix} -384{,}04 \\ 45{,}77 \\ 242{,}22 \\ 96{,}04 \end{bmatrix}$

Using the initial conditions:

$S_0 = 15.783.325,$
$E_0 = 1.676,$
$I_0 = 838,$
$R_0 = 0,$

Thus, $y_1 = \begin{bmatrix} 15.783.325 \\ 1.676, \\ 838 \\ 0 \end{bmatrix} + \begin{bmatrix} -384{,}04 \\ 45{,}77 \\ 242{,}22 \\ 96{,}04 \end{bmatrix} = \begin{bmatrix} 15.782.940{,}96 \\ 1.721{,}77 \\ 1.080{,}22 \\ 96{,}04 \end{bmatrix}$

Since the SEIR model consists of four compartments, the above equation can be expressed in the following form:

$$\begin{bmatrix} S_1 \\ E_1 \\ I_1 \\ R_1 \end{bmatrix} = \begin{bmatrix} S_0 \\ E_0 \\ I_0 \\ R_0 \end{bmatrix} + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

together with the values of k_1, k_2, k_3, dan k_4 Using the values obtained from MATLAB, the compartment values for the first iteration are obtained as follows:

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

https://doi.org/10.30605/proximal.v9i2.9552

Table 2. Results of the First Iteration Using the Fourth-Order Runge–Kutta Method

Compartement	values
S_1	15.782.940,96
E_1	1.721,77
I_1	1.080,22
R_1	96,04

he values presented in Table 4.2 were obtained from numerical computations performed in MATLAB based on Equation (10).

Simulation Results of the Fourth-Order Runge–Kutta Method

The subsequent computations were performed iteratively using the Fourth-Order Runge–Kutta (RK4) algorithm up to the 100 th iteration with a step size of $(h = 1)$. The computational process was carried out using MATLAB. The simulation results are presented in **Table 3**.

Table 3. Numerical Simulation Results of the SEIR Model Using the Fourth-Order Runge Kutta Method

Day	Susceptible	Exposed	Infected	Recovered
0	15783325	1676	838	0
1	15782941	1721.77	1080.22	96.04246
10	15774318	5031.611	4236.169	2252.825
20	15738053	19763.5	16685.78	11336.31
30	15596499	77031.58	65315.15	46993.12
40	15060221	290286	250253.1	185078.7
50	13239735	966604.2	885975	693525
60	8857818	2200600	2447643	2279778
70	3805583	2355844	4009738	5614675
80	1391500	1190597	3613947	9589796
90	667900	418617.3	2198302	12501020
100	448456.1	137675.7	1099318	14100389

Based on Table 3, the number of individuals in the Susceptible (S) compartment continuously decreases, while the Exposed (E), Infected (I), and Recovered (R) compartments change according to the dynamics of disease transmission. The simulation results indicate that the Fourth-Order Runge–Kutta method is able to produce numerical solutions that describe the dynamics of measles transmission in the SEIR model.

Simulation Results Graph of the Fourth-Order Runge–Kutta Method

The changes in the number of individuals in each compartment are illustrated through the simulation results obtained using the Fourth-Order Runge–Kutta method. The simulation results are presented in graphical form. The graph shows the changes in the number of individuals in the Susceptible (S), Exposed (E), Infected (I), and Recovered (R) compartments over the 100-day simulation period.

ISSN 26158132 (cetak)
ISSN 26157667 (online)

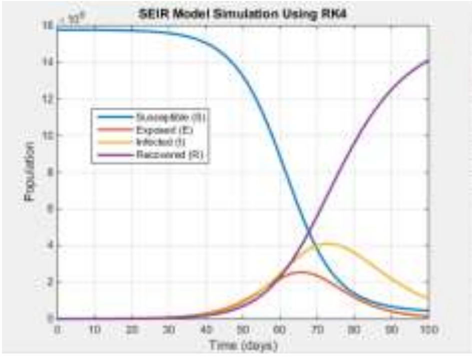

Figure 2. Simulation Results Using the Runge–Kutta Method

Based on Figure 2, the number of individuals in the Susceptible (S) compartment continuously decreases, while the Exposed (E) and Infected (I) compartments increase at the beginning of the simulation before decreasing after reaching their peaks. In contrast, the number of individuals in the Recovered (R) compartment continuously increases until the end of the simulation. These results indicate that the Fourth-Order Runge–Kutta method is capable of describing the dynamics of measles transmission in the SEIR model throughout the simulation period.

The Effect of the Control Variable on Measles Transmission

The simulation was conducted to analyze the effect of the control variable on the dynamics of measles transmission using the SEIR model. The numerical solution was obtained using the Fourth-Order Runge–Kutta (RK4) method with a step size of $(h = 1)$ day over a period of 100 days. The applied control variable consists of four scenarios, namely $(u = 0), (u = 0.3), (u = 0.6),$ and $(u = 0.9)$. The simulation results are presented in graphical form to compare the changes in the number of individuals in each compartment (Susceptible, Exposed, Infected, and Recovered) under different control levels.

Simulation Results with Control Value (u = 0)

The control value $(u = 0)$ represents a condition without any control intervention in measles transmission. Under this condition, the transmission rate occurs naturally according to the model parameters without any effort to reduce contact between susceptible and infected individuals. The SEIR model simulation results with the control value $(u = 0)$ are shown in Figure 3.

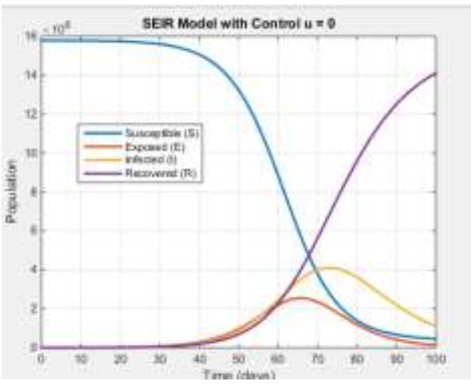

Figure 3. SEIR Model with $(u = 0)$

Based on Figure 3, under the uncontrolled condition $(u = 0)$,the number of individuals in the Susceptible (S) compartment continuously decreases, while the Exposed (E) and Infected (I) compartments increase at the beginning of the simulation before decreasing due to the recovery process. In contrast, the number of individuals in the Recovered (R) compartment continuously increases throughout the simulation period. The simulation results show that without control intervention, measles transmission occurs more rapidly and produces a higher

ISSN 26158132 (cetak)
ISSN 26157667 (online)

number of infected individuals. Therefore, this condition is used as a comparison to evaluate the effectiveness of applying the control variable in the subsequent scenarios.

Simulation Results with Control Value $(u = 0.3)$

The control value (u = 0.3) indicates that a 30% control intervention is applied to the measles transmission rate. The implementation of this control reduces the probability of effective contact between susceptible and infected individuals compared to the uncontrolled condition. The SEIR model simulation results with the control value (u = 0.3) are shown in Figure 4.

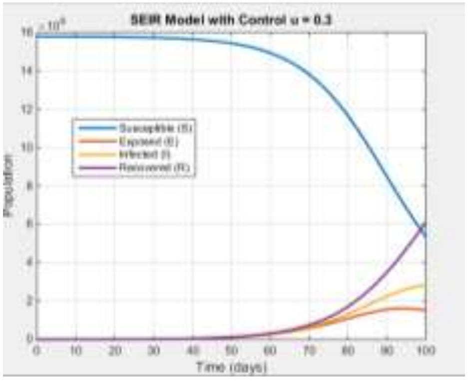

Figure 4. SEIR Model with $(u = 0.3)$

Based on **Figure 4**, the implementation of a control variable of $(u = 0.3)$ causes the decrease in the number of individuals in the Susceptible (S) compartment to occur more slowly compared to the uncontrolled condition. In addition, the number of individuals in the Exposed (E) and Infected (I) compartments is also lower, while the Recovered (R) compartment continues to increase. The simulation results indicate that a 30% control intervention is able to reduce the transmission rate of measles, although it has not yet optimally suppressed the number of infected individuals.

Simulation Results with Control Value $(u = 0.6)$

The control value $(u = 0.6)$ indicates that a 60% control intervention is applied to the measles transmission rate. Under this condition, the probability of effective contact between susceptible and infected individuals is further reduced compared to the conditions of $(u = 0)$and $(u = 0.3)$. The SEIR model simulation results with the control value $(u = 0.6)$ are shown in Figure 5.

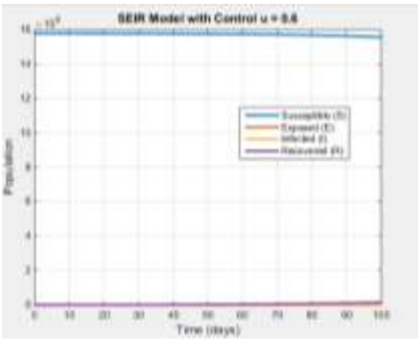

Figure 5. SEIR Model with $(u = 0.6)$

Based on **Figure 5**, the implementation of a control variable of $(u = 0.6)$ causes the decrease in the number of individuals in the Susceptible (S) compartment to occur more slowly compared to the scenarios of $(u = 0)$ and $(u = 0.3)$. The number of individuals in the Exposed (E) and Infected (I) compartments also decreases more significantly, while the Recovered (R) compartment continues to increase. The simulation results indicate that a 60% control intervention is more effective in reducing measles transmission compared to the previous two

https://doi.org/10.30605/proximal.v9i2.9552

scenarios, as indicated by the lower number of infected individuals and the reduced rate of transition from the Susceptible compartment to the Exposed compartment.

Simulation Results with Control Value ($u = 0.9$)

The control value ($u = 0.9$)indicates that a 90% control intervention is applied to the measles transmission rate. Under this condition, the probability of effective contact between susceptible and infected individuals becomes very small, allowing disease transmission to be maximally reduced. The SEIR model simulation results with the control value ($u = 0.9$) are shown in Figure 6.

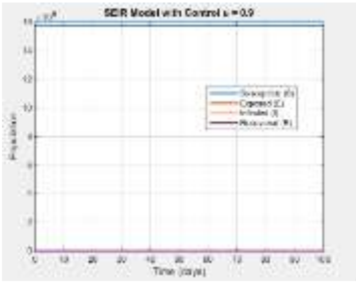

Figure 6. SEIR Model with ($u = 0.9$)

Based on **Figure 6**, the implementation of a control variable of ($u = 0.9$) results in the smallest decrease in the number of individuals in the Susceptible (S) compartment compared to the scenarios of ($u = 0$), ($u = 0.3$), and ($u = 0.6$). The number of individuals in the Exposed (E) and Infected (I) compartments is also the lowest, while the Recovered (R) compartment continues to increase. The simulation results indicate that a 90% control intervention is the most effective scenario for reducing measles transmission. The higher the value of the applied control variable, the more effective the control strategy is in suppressing disease transmission in the SEIR model.

D. Conclusion

Based on the results of this study, the SEIR (Susceptible–Exposed–Infected–Recovered) model is able to describe the dynamics of measles transmission in North Sumatra through changes in the number of individuals in each compartment. The solution of the differential equation system using the Fourth-Order Runge–Kutta (RK4) method produced stable numerical solutions throughout the 100-day simulation period with a step size of (h = 1) day. The simulation results also show that the application of the control variable has an effect on reducing disease transmission, as indicated by the decreasing number of individuals in the Exposed (E) and Infected (I) compartments and the increasing number of individuals in the Susceptible (S) compartment as the control value increases. Among the tested scenarios, the control value (u = 0.9) represents the most effective scenario in suppressing measles transmission in the model used in this study.

REFERENCES

Asy-syifaa, Kurniasih, A., Lubis, S. A. L., & Damanik, R. Z. (2024). Karakteristik Kejadian Campak Pada Anak Di Rsud Dr. Fauziah Kabupaten Bireuen Tahun 2022. *Jurnal Kedokteran STM (Sains Dan Teknologi Medik)*, *VII*(I), 24–32.

Beni, O., Anisa, K., Cahyanti, N., Nisa, R., & Agushybana, F. (2021). Dampak Revolusi Mobilisasi Penduduk terhadap Persebaran Penyakit Menular di Indonesia. *Jurnal Kesehatan Masyarakat*, *7*(1).

Chebotaeva, V., Srinivasan, A., & Vasquez, P. A. (2025). Differentiating Contact with Symptomatic and Asymptomatic Infectious Individuals in a SEIR Epidemic Model. *Mathematical Biology*, *87*(38), 1–25.

Costa, A. L. P. da, Pires, M. A., Resque, R. L., & Almeida, and S. S. M. da S. de. (2021). Mathematical Modeling of the Infectious Diseases: Key Concepts and Applications. *Infectious Diseases and Epidemiology*, *7*(5), 1–9.

Lim, D., Ko, K. T., Hong, H., Lee, H., & Choi, B. (2025). *A history-dependent approach for accurate initial condition estimation in epidemic models*. 1–21. https://doi.org/10.1371/journal.pcbi.1013438

Nisa, S. F., Yulida, Y., Keahlian, K., & Lingkungan, M. (2022). Modifikasi Model Seir Pada Penyakit Campak. *Jurnal Matematika Murni Dan Terapan*, *16*(1), 13–27.

Pasaribu, S. M., & Husein, I. (2024). Modeling The Proportion of Measles Cases in North Sumatra by The Jack-Knife Partial Least Squares Method. *Journal of Computer Science, Information Technology and Telecommunication Engineering (JCoSITTE)*, *5*(2), 669–678. https://doi.org/10.30596/jcositte.v5i2.20928

Vitanov, N. K. (2023). Computation of the Exact Forms of Waves for a Set of Differential Equations Associated with the SEIR Model of Epidemics. *Computation*, *11*(129), 1–10.

Xerri, T., Darmanin, N., Zammit, M. A., & Fsadni, C. (2020). Complications of measles : a case series. *BMJ Publishing Group Limited*, *13*(3), 3–6. https://doi.org/10.1136/bcr-2019-232408

Zahrah, N. D., Nurani, F. S., Amanda, A. P., Muthia, F., & Herbawan, C. K. (2023). STUDI LITERATUR: ANALISIS FAKTOR RISIKO CAMPAK PADA ANAK DI INDONESIA. *Medika Malahayati*, *7*(3), 748–758.