Proximal: Jurnal Penelitian Matematika dan Pendidikan Matematika
ISSN 26158132 (cetak)
ISSN 26157667 (online)

STATISTICAL PROCESS CONTROL WITH REAL-WORLD WATER QUALITY DATA: AN ILLUSTRATIVE MEC CONTROL CHART APPROACH FOR MATHEMATICAL SCIENCE

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Abstrak. Pengendalian Proses Statistik (SPC) merupakan topik penting dalam statistik terapan dan ilmu matematika, terutama dalam konteks pemantauan stabilitas proses dan deteksi pergeseran kecil pada parameter proses. Meskipun metode SPC banyak dibahas dalam literatur teknik kualitas, aplikasi ilustratif menggunakan data dunia nyata tetap berharga untuk memperkuat pemahaman konseptual dalam jurnal statistik dan matematika. Artikel ini menyajikan aplikasi ilustratif SPC menggunakan pengukuran klorin sisa dari proses produksi air. Analisis ini menggunakan diagram kendali Mixed Exponentially Weighted Moving Average–Cumulative Sum (MEC) yang dilengkapi dengan fitur Fast Initial Response (FIR), yaitu Basic FIR (BFIR) dan Modified FIR (MFIR). Hasil menunjukkan bahwa diagram MEC yang dilengkapi dengan fitur MFIR secara konsisten menghasilkan batas kendali yang lebih sempit selama fase pemantauan awal dan mendeteksi penyimpangan dini dengan lebih efektif dibandingkan diagram berbasis BFIR. Studi ini memberikan contoh terapan dan jelas secara konseptual tentang metodologi SPC lanjutan menggunakan data lingkungan autentik, yang dapat menjadi acuan bagi pembaca di bidang statistik terapan dan ilmu matematika.

Kata Kunci: Pengendalian Kualitas Statistik; Statistik Terapan; Diagram Pengendalian MEC; Respons Awal Cepat; Data Kualitas Air.

Abstract. Statistical Process Control (SPC) constitutes an important topic in applied statistics and mathematical science, particularly in the context of monitoring process stability and detecting small shifts in process parameters. While SPC methods are widely discussed in the quality engineering literature, illustrative applications using real-world data remain valuable for strengthening conceptual understanding in statistics and mathematics-oriented journals. This paper presents an illustrative application of SPC using residual chlorine measurements from a water production process. The analysis employs Mixed Exponentially Weighted Moving Average–Cumulative Sum (MEC) control charts enhanced with Fast Initial Response (FIR) features, namely Basic FIR (BFIR) and Modified FIR (MFIR). The results show that MEC charts incorporating MFIR features consistently produce narrower control limits during the initial monitoring phase and detect early deviations more effectively than BFIR-based charts. This study contributes an applied and conceptually clear example of advanced SPC methodology using authentic environmental data, which may serve as a reference for applied statistics and mathematical science audiences.

Keywords: Statistical Process Control; Applied Statistics; MEC Control Charts; Fast Initial Response; Water Quality Data.

A. Introduction

Statistical Process Control (SPC) plays a significant role in applied statistics and mathematical science, particularly in understanding and monitoring process variation and detecting departures from stochastic stability (Montgomery, 2009). Classical Shewhart control charts are simple to implement and widely used; however, they are known to be less sensitive to small or gradual shifts in the process mean because they rely solely on the most recent observation (Roberts, 1959)(Shewhart, 1929). To address this limitation, alternative approaches such as the Cumulative Sum (CUSUM) and Exponentially Weighted Moving Average

(EWMA) control charts have been extensively developed and applied in both theoretical and applied contexts (Page, 1954).

CUSUM charts accumulate deviations from a target value to enhance sensitivity to sustained mean shifts (Lucas & Crosier, 1982), while EWMA charts incorporate historical information through exponential weighting, allowing earlier detection of small changes in process parameters (Lucas & Saccucci, 1990). Both methods are well documented to outperform Shewhart charts in detecting small shifts in process parameters (Montgomery, 2009). Further methodological developments have focused on improving robustness and interpretability through hybrid approaches that integrate multiple charting schemes. One such development is the Mixed EWMA–CUSUM (MEC) control chart, which combines the memory properties of EWMA with the cumulative structure of CUSUM, offering a flexible framework for mean shift detection (Mohammadkhani & Amiri, 2022).

Recent studies in applied statistics have continued to explore hybrid and adaptive control chart frameworks to address challenges associated with small shift detection and start-up monitoring in complex processes (Alevizakos, Chatterjee, & Koukouvinos, 2021)(Devianto et al., 2024)(Talordphop, Sukparungsee, & Areepong, 2023). These studies demonstrate that extensions of classical SPC methods, including hybrid EWMA–CUSUM structures and dynamic control limit adjustments, remain an active area of methodological development and applied research.

In addition to hybrid chart designs, Fast Initial Response (FIR) features have been introduced to modify the behavior of control charts during the start-up phase of monitoring or immediately after a process reset (Lucas & Crosier, 1982)(Steiner, 1999). FIR features dynamically adjust control limits in the early stages of monitoring, thereby influencing early-stage chart behavior without altering the underlying monitoring statistic (Rhoads, Montgomery, & Mastrangelo, 1996)(Chiu, 2009).

Although MEC charts and FIR features have been widely studied in quality engineering contexts, illustrative applications using real-world environmental data remain relatively limited in the applied statistics literature. This paper addresses this gap by presenting an illustrative application of MEC control charts with FIR features using residual chlorine data from a real-world water quality monitoring context. Residual chlorine was specifically selected as the monitoring variable because it is a sequentially collected, univariate measurement with a clearly defined in-control target, making it well-suited for illustrating time-ordered SPC methods. Moreover, as a regulated parameter governing microbiological safety in treated water, it provides an authentic and practically meaningful process context for this illustrative analysis. (Asmara et al., 2021)(Asmara et al., 2021)(Kwio-tamale & Onyutha, 2024). From a mathematical and applied statistics perspective, the study emphasizes conceptual understanding of smoothing mechanisms, cumulative monitoring, and dynamic control limit behavior within a unified SPC framework. The objective is not to propose a new control chart, but to provide a clear and applied demonstration of how advanced SPC methods behave when applied to authentic data. From an applied mathematics and mathematical science perspective, this study emphasizes the structural behavior of hybrid control charts, focusing on the interaction between smoothing mechanisms, cumulative monitoring, and dynamic control limit adjustment rather than on domain-specific operational optimization.

B. Material And Method

1. Data Description

The dataset analyzed in this study consists of univariate measurements of free residual chlorine collected sequentially from a regional water production process. Residual chlorine is a critical water quality parameter used to ensure microbiological safety in treated water. From a statistical perspective, the data form a natural time-ordered process suitable for illustrating

SPC techniques. The observations are treated as individual measurements monitored over time, with the process target and variability estimated from the in-control period. No data transformation or artificial manipulation is applied, ensuring that the analysis reflects realistic process behavior and variability.

2. Statistical Process Control Methods

This subsection presents the main statistical formulations of the control charts used in this study using inline mathematical notation. The objective is to ensure mathematical clarity while maintaining readability for applied statistics and mathematical science audiences. Detailed derivations and optimal parameter design are omitted, as they are well established in the literature.

a. Exponentially Weighted Moving Average (EWMA)

The EWMA control chart is designed to detect small and moderate shifts in the process mean by incorporating past observations through exponential weighting. The smoothing parameter governs the balance between sensitivity and stability, with smaller values providing stronger memory of historical data (Roberts, 1959)(Letshedi et al., 2021). The EWMA statistic is defined as:

$$Z_i = \lambda x_i + (1-\lambda)Z_{i-1}$$

where x_i denotes the observation at time i, $\lambda \in (0,1]$ is the smoothing parameter, and the initial value Z_0 is typically set equal to the in-control process mean μ_0. The corresponding control limits at time i are given by:

$$UCL\ or\ LCL = \mu_0 \pm L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-\left(1-\lambda_q\right)^{2i}\right]}$$

where σ denotes the in-control standard deviation and L is a width constant.

b. Cumulative Sum (CUSUM)

Cumulative Sum (CUSUM) control charts monitor cumulative deviations from a specified target value and are effective for detecting sustained shifts in the process mean. The one-sided tabular CUSUM statistics are defined as:

$$C_i^+ = (x_i - \mu_0) - K + C_{i-1}^+,$$
$$C_i^- = -(x_i - \mu_0) - K + C_{i-1}^-$$

where K is the reference value controlling sensitivity. An out-of-control signal is generated when $C_i^+ > h$ or $C_i^- > h$, with h denoting the decision interval (Page, 1954)(Hu et al., 2022).

c. Mixed EWMA–CUSUM (MEC)

The Mixed EWMA–CUSUM (MEC) control chart integrates the EWMA statistic into the CUSUM framework, combining the memory property of EWMA with the cumulative structure of CUSUM (Osei-Aning, Abbasi, & Riaz, 2017)(Mohammadkhani & Amiri, 2022). Let Z_i denote the EWMA statistic. The MEC monitoring statistics are defined as:

$$M_t^+ = max\{0, M_{\{t-1\}}^+ + (Z_t - \mu_0 - k)\}$$
$$M_t^- = max\{0, M_{\{t-1\}}^- + (\mu_0 - Z_t - k)\}$$

with initial values $M_0^+ = M_0^- = 0$. Control limits are determined using a predefined decision interval h. This hybrid structure enhances sensitivity to small and moderate shifts while retaining interpretability.

d. Fast Initial Response (FIR) Features

Fast Initial Response (FIR) features improve detection sensitivity during the initial monitoring phase by dynamically adjusting control limits without altering the monitoring statistics (Lucas & Crosier, 1982)(Sari, Maiyastri, & Devianto, 2024). The Basic Fast Initial

https://doi.org/10.30605/proximal.v9i1.7860

Response (BFIR) adjusts the decision interval according to $h_t = h[1 - f(1 - r^t)]$, where $f(0 < f < 1)$ controls the degree of initial tightening and $r(0 < r < 1)$ determines the rate at which the control limits converge to their standard values. The Modified Fast Initial Response (MFIR) applies a stronger initial adjustment, resulting in narrower control limits during the start-up phase. Both FIR features converge to the standard MEC control limits as monitoring progresses.

C. Hasil Penelitian dan Pembahasan

1. Results and Illustrative Analysis

This section presents an illustrative comparison of Mixed EWMA–CUSUM (MEC) control charts incorporating different Fast Initial Response (FIR) features when applied to real-world residual chlorine data. The analysis is intended to demonstrate the behavior of the control charts under representative parameter settings rather than to provide an exhaustive performance evaluation. Because control charts inherently represent time-ordered observations, the illustrative analysis focuses directly on the control charts without presenting the raw time series. The dataset used in this analysis consists of sequential residual chlorine measurements sourced from Asmara et al. (2021) and Kwio-tamale & Onyutha (2024). All computations and graphical illustrations were performed using the R statistical software with custom implementations of the MEC statistic and FIR adjustment functions following the formulations described in Section 2.

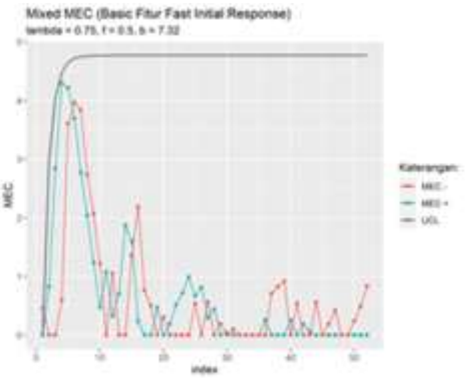

Figure 1. MEC control chart with BFIR features applied to residual chlorine data.

Figure 1 shows the MEC control chart with Basic Fast Initial Response (BFIR) features applied to the residual chlorine dataset. The control limits exhibit a wider range during the initial monitoring phase, reflecting the more conservative adjustment associated with the BFIR mechanism. Under this configuration, several observations approach the control limits, illustrating how the chart responds to natural process variability during early monitoring.

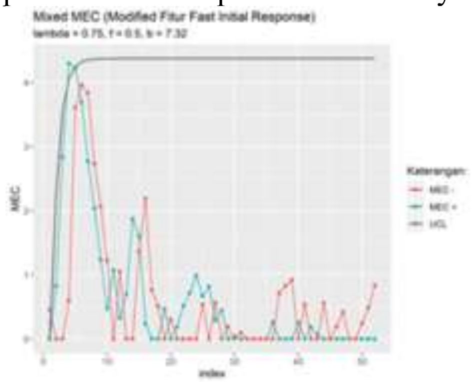

Figure 2. MEC control chart with MFIR features applied to the same dataset and parameter settings as Figure 1.

Figure 2 presents the MEC control chart with Modified Fast Initial Response (MFIR) features applied to the same dataset using identical parameter settings. Compared with the BFIR-based chart in Figure 1, the MFIR-enhanced MEC chart displays visibly narrower control limits at the beginning of the monitoring period. This difference illustrates the stronger initial adjustment imposed by the MFIR feature and highlights its influence on early-stage chart behavior.

Across the illustrated configurations, the BFIR-based MEC charts tend to maintain wider control limits during the start-up stage, reflecting a more conservative initial adjustment. In contrast, the MFIR-based charts exhibit narrower control limits over the same period, with several observations located closer to the control boundaries. These differences are more apparent for larger values of the FIR adjustment parameter, where the effect of initial control limit modification becomes visually pronounced. As monitoring progresses, the control limits under both BFIR and MFIR features gradually converge toward the standard MEC control limits. This convergence behaviour is consistent with the theoretical properties of FIR mechanisms, which are designed to influence only the early phase of monitoring without altering long-term control chart characteristics.

The results illustrate how FIR features affect the early-stage behaviour of MEC control charts rather than their long-run properties. From an applied statistics perspective, this illustrative analysis highlights the interaction between smoothing, cumulative monitoring, and dynamic control limit adjustment within a unified SPC framework when applied to authentic environmental data. Overall, the graphical illustrations provided by the MEC control charts with BFIR and MFIR features offer a clear comparison of early-stage monitoring behavior under identical data and parameter settings. Rather than emphasizing numerical performance measures, the results focus on visual and structural differences in control limit evolution and signal behavior. This illustrative approach allows readers to directly observe how FIR features influence chart behavior in practice, thereby supporting conceptual understanding of hybrid SPC methods when applied to real-world data.

2. Discussion

The illustrative analysis demonstrates that incorporating Fast Initial Response (FIR) features into Mixed EWMA–CUSUM (MEC) control charts influences the behavior of the charts during the early stage of monitoring. In particular, differences in the width of the initial control limits reflect how alternative FIR mechanisms adjust the start-up response of the monitoring scheme. From an applied statistics perspective, these results highlight the interaction between smoothing mechanisms, cumulative monitoring, and dynamic control limit adjustment within a unified SPC framework. The observed convergence of control limits over time is consistent with the theoretical design of FIR features, which aim to modify early-stage behavior without affecting long-run chart properties. This study emphasizes methodological illustration rather than operational decision-making or performance optimization. By applying advanced SPC techniques to real-world water quality data, the analysis provides a concrete and interpretable example that supports conceptual understanding of hybrid control charts and their statistical behavior under realistic process variability. Beyond its applied context, the illustrative analysis presented in this study may also serve as a conceptual reference for higher-level mathematics and statistics education, where understanding the behavior of monitoring statistics is an essential component of applied mathematical training.

From a methodological standpoint, the illustrative results highlight how different design components of SPC charts interact to influence early monitoring behaviour. The combination of smoothing mechanisms inherent in EWMA statistics and cumulative structures from

CUSUM charts provides a flexible monitoring framework that balances responsiveness and stability. The inclusion of FIR features further demonstrates how temporary control limit adjustment can be used to emphasize early-stage detection without modifying the underlying monitoring statistic.

Importantly, this illustrative analysis shows that differences between BFIR and MFIR features are primarily manifested during the start-up phase, while long-run behaviour remains consistent with standard MEC control charts. This observation reinforces the conceptual role of FIR mechanisms as transient enhancements rather than permanent structural changes. Such insights are particularly relevant for applied statistics and mathematical science audiences, where understanding the behaviour of statistical procedures under different design configurations is as important as operational performance.

D. Conclusion

This paper presents an illustrative application of Mixed EWMA–CUSUM (MEC) control charts incorporating Fast Initial Response (FIR) features using real-world residual chlorine data. The analysis demonstrates how alternative FIR mechanisms influence the behavior of MEC control charts during the initial phase of monitoring, particularly through differences in the evolution of control limits.

The contribution of this study lies in providing a clear and applied illustration of advanced SPC methodology using authentic environmental data. By emphasizing methodological behavior rather than performance optimization, the findings offer a useful reference for researchers and readers in applied statistics and mathematical science who seek to understand the practical characteristics of hybrid control charts under realistic process variability. Beyond the specific application presented, the illustrative framework adopted in this study may be extended to other univariate monitoring contexts where understanding the interaction between smoothing, accumulation, and dynamic control limits is of interest. In addition to its contribution to applied statistics, this illustrative framework can support the teaching and learning of advanced statistical process control methods within applied mathematics and mathematical science contexts.

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