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MATHEMATICAL TOPOLOGY MEETS TRADITION: ALEXANDER POLYNOMIAL ANALYSIS OF SIDALUNGGUH KETUPAT WEAVING PATTERNS

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Abstrak. Penelitian ini bertujuan untuk mendeskripsikan hubungan antara matematika dan budaya kuliner, khususnya pada ketupat, khususnya ketupat Sidalungguh. Dengan menggunakan aplikasi teori simpul, penelitian ini mengkaji bagaimana matematika dan budaya kuliner saling terkait. Analisis ini dilakukan dengan membandingkan simpul pada ketupat dengan literatur teori simpul, yang mengarah pada pembuatan diagram simpul. Melalui identifikasi menggunakan polinomial Alexander, diperoleh hasil sebagai berikut:

$$\Delta K = t^{12} - 9t^{11} + 25t^{10} - 25t^{9} + 9t^{8} - t^{7}$$

Penelitian ini dilakukan untuk mengeksplorasi potensi ilmiah dalam mengkaji matematika melalui makanan tradisional.

Keywords: **Topologi, Teori Knot, Alexander Polynomial, Culture, Sidalungguh Ketupat**

Abstrak. This research aims to describe the relationship between mathematics and culinary culture, specifically focusing on ketupat, particularly the Sidalungguh Ketupat. Using knot theory applications, this study examines how mathematics and culinary culture are interconnected. This analysis was conducted by comparing the knots in the ketupat with knot theory literature, leading to the creation of knot diagrams. Through identification using Alexander polynomials, the following result was obtained:

$$\Delta K = t^{12} - 9t^{11} + 25t^{10} - 25t^{9} + 9t^{8} - t^{7}$$

This research was conducted to explore the scientific potential in examining mathematics through traditional food.

Kata Kunci: **Topology, Knot Theory, Alexander Polynomial, Culture, Sidalungguh Ketupat**

A. Introduction

Recent studies indicate that mathematical education faces challenges in connecting abstract theoretical concepts with real-world applications, particularly in topology and knot theory(Audun Holme, 2012; Buck, 2009; D.W. Sumners, 2007; SIDNEY A. MORRIS, 2014). This issue has significant implications for both mathematics education and cultural preservation. While researchers have investigated the theoretical aspects of topology and knot theory extensively, there remains a critical gap in understanding how these mathematical principles manifest in traditional cultural artifacts.

Previous work by Johar et al., (2023) and Shirawia et al. (2023) demonstrated that mathematics serves as a foundational discipline intrinsically linked to logical reasoning and scientific understanding. Pinter Pandai (2018) Morris (2007) established Ja'faruddin & Chen, (2024) the fundamental principles of topology as a branch examining spatial properties invariant under continuous two-way deformation, introducing the concept of "rubber-sheet geometry." Additionally, Ja'faruddin & Haw (2024) revealed the significance of the Alexander polynomial in analyzing knot structures and their properties.

This study aims to investigate the topological properties and knot theoretical aspects of ketupat Sidalungguh construction by employing a mixed-method approach combining mathematical analysis and ethnographic research. Our methodology involves:

1. Mathematical modeling of katupat Sidalungguh weaving patterns
2. Application of Alexander polynomial analysis
3. Documentation of traditional weaving techniques
4. Correlation analysis between mathematical properties and cultural significance

We hypothesize that the traditional construction methods of katupat incorporate sophisticated mathematical principles of knot theory, which can be quantified and analyzed using modern topological tools based on the theoretical framework of Alexander polynomials and invariant theory.

The findings will contribute to:

1. Theoretical mathematics: Expanding the application scope of knot theory
2. Cultural preservation: Documenting and analyzing traditional crafting techniques
3. Mathematics education: Providing concrete examples of abstract mathematical concepts

This research will provide practical insights for:

a. Mathematics educators seeking real-world applications of topology
b. Cultural preservationists documenting traditional crafting methods
c. Students studying advanced mathematics
d. Craftspeople and artisans working with traditional weaving techniques

This research is particularly significant because it bridges the gap between abstract mathematical theory and cultural heritage, offering a unique perspective on how traditional knowledge systems inherently incorporate advanced mathematical concepts. The integration of topology and knot theory with cultural artifacts not only preserves traditional knowledge but also demonstrates the universal nature of mathematical principles across different cultures and contexts.

B. Reserach Methodology

The research methodology for this study adopts a qualitative exploratory design that incorporates mathematical analysis, with a specific focus on applying knot theory and Alexander polynomials to traditional artifacts. The primary subject of investigation is the Sidalungguh ketupat, a traditional woven food casing from Sidamukti Village, Sidareja District, Cilacap, Central Java, Indonesia. The study employs purposive sampling in selecting this particular ketupat variant due to its distinctive knot-based structural components.

Data collection encompasses both primary and secondary sources. Primary data includes visual documentation of ketupat construction, structural analysis of knot patterns, and mathematical modeling of identified knots. Secondary data comprises historical documentation, cultural significance records, and previous studies on traditional weaving patterns. The research process is structured into four distinct phases, beginning with a preliminary analysis that involves specimen selection, documentation, and initial visual examination of knot patterns.

The second phase focuses on knot identification and analysis, incorporating detailed visual inspection of knot configurations, comparative analysis with established knot theory literature, development of knot diagrams, and classification of identified knot types. The third phase delves into mathematical analysis, where mathematical models are constructed for identified knots, Alexander polynomial calculations are applied, and unique knot characteristics are documented. The fourth phase ensures validation and verification through cross-referencing with existing literature, mathematical calculation verification, and expert consultation.

The data analysis framework follows a comprehensive approach that combines qualitative and mathematical methods (Michael Angrosino, 2557; Parratt, 2000; Paul Atkinson, Amanda

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Coffey, Sara Delamont, 2557; Plomp, 2013; Taylor & Medina, 2013). Qualitative analysis includes pattern recognition in knot structures, historical context interpretation, and cultural significance assessment. Mathematical analysis focuses on Alexander polynomial computation, knot invariant analysis, and structural property evaluation. These analyses are then synthesized to integrate qualitative and mathematical findings, conduct pattern correlation analysis, and develop a theoretical framework.

To maintain research quality and reliability, several measures are implemented, including triangulation of data sources, expert verification of mathematical calculations, comprehensive documentation of all procedures, and multiple iterations of analysis for consistency. The study acknowledges certain limitations, including sample size constraints, regional specificity of the artifact, and mathematical model approximations. This methodological approach ensures a systematic investigation while maintaining scientific rigor appropriate for topological analysis of cultural artifacts, allowing for both mathematical precision and preservation of cultural context in analyzing traditional weaving patterns.

C. Result and Discussion

The Sidalungguh ketupat represents a distinctive culinary heritage from Sidamukti Village, Sidareja District, Cilacap, Central Java, Indonesia. This traditional food item is renowned for its distinctive octagonal shape with prominent vertices. The origin of Sidalungguh ketupat is deeply rooted in local folklore, centered around Princess Dewi Sidalungguh, who possessed mystical powers to create an endless supply of ketupat. According to legend, she used this ability to feed her people during times of natural disasters and famine, making the ketupat not just a food item but a symbol of sustenance and divine providence.

Figure 1. Ketupat Sidalungguh

The Sidalungguh ketupat has evolved into a powerful symbol of prosperity, welfare, and divine blessing in local culture. Local residents have integrated it into their traditional ceremonies, celebrations, and festivals, making it an integral part of their cultural heritage.

In its preparation, Sidalungguh ketupat is crafted using glutinous rice wrapped in young coconut leaves (janur) and boiled until cooked. The distinctive coloring process and techniques employed in its creation add to its unique characteristics. Beyond its unique form and historical background, the Sidalungguh ketupat transcends its role as mere traditional food, embodying local wisdom and the rich cultural heritage of Central Java. Its spiritual and cultural values make it a significant symbol of regional identity.

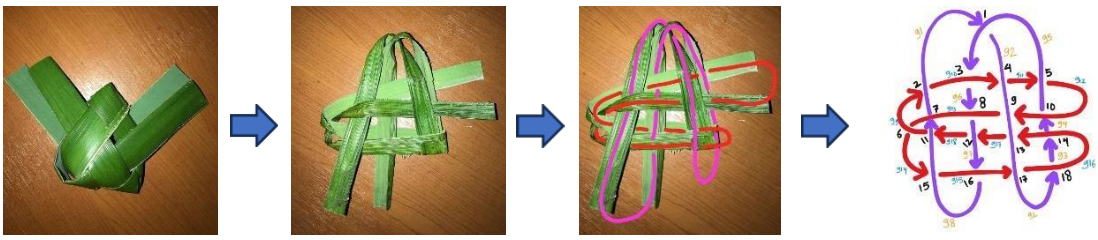

Figure 2 Transformation of Ketupat Sidalungguh

In calculating the knot structure, this study employed both right-hand and left-hand crossing rules, analyzing 18 knot points and 18 crossing points, each uniquely labeled. This systematic approach follows standard knot theory protocols, where each crossing point is carefully identified and classified according to its orientation. The labeling system enables precise tracking of how the strands interweave, creating the distinctive octagonal pattern characteristic of the Sidalungguh ketupat. This methodical analysis provides a foundation for applying the Alexander polynomial calculations to quantify the knot's topological properties.

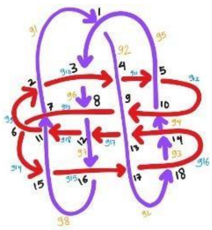

Figure 3. Knot Diagram of Sidalungguh Ketupat

The topological properties of the Sidalungguh ketupat's knot structure can be confirmed through matrix determinant calculations based on the knot diagram shown in Figure 4.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
G1	-1	1-t	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0
G2	t	0	0	1-t	0	0	0	0	1-t	0	0	0	1-t	0	0	0	1-t	-1
G3	0	0	0	0	0	0	0	0	0	0	0	0	0	t	0	0	0	t
G4	0	0	0	0	0	0	0	0	0	t	0	0	0	-1	0	0	0	0
G5	1-t	0	t	0	1-t	0	0	0	0	-1	0	0	0	0	0	0	0	0
G6	0	0	-1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0
G7	0	0	0	0	0	0	0	t	0	0	0	1-t	0	0	0	t	0	0
G8	0	0	0	0	0	0	t	0	0	0	1-t	0	0	0	1-t	-1	0	0
G9	0	t	0	0	0	1-t	0	0	0	0	t	0	0	0	0	0	0	0
G10	0	-1	1-t	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
G11	0	0	0	t	t	0	0	0	0	0	0	0	0	0	0	0	0	0
G12	0	0	0	0	-1	0	0	0	t	1-t	0	0	0	0	0	0	0	0
G13	0	0	0	0	0	-1	1-t	1-t	-1	0	0	0	0	0	0	0	0	0
G14	0	0	0	0	0	t	0	0	0	0	0	0	0	0	t	0	0	0
G15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1-t	-1	0
G16	0	0	0	0	0	0	0	0	0	0	0	0	t	1-t	0	0	t	1-t
G17	0	0	0	0	0	0	0	0	0	0	0	t	-1	0	0	0	0	0
G18	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0

Figure 4 Matriks Alexander for Ketupat Sidalungguh

To determine the Alexander polynomial for the Sidalungguh ketupat, we employ the matrix calculation procedure

The calculation of the Alexander polynomial for the Sidalungguh ketupat begins with matrix reduction, where we select a row and column for deletion to create an (17)×(17) matrix. The determinant is then calculated through various methods including cofactor expansion, Laplace expansion, and Gaussian elimination with polynomial entries. Following this, the polynomial undergoes normalization by dividing by the highest power of t and ensuring polynomial symmetry with appropriate coefficient normalization.

The resulting Alexander polynomial for the Sidalungguh ketupat is expressed as $\Delta K = t^{12} - 9t^{11} + 25t^{10} - 25t^{9} + 9t^{8} - t^{7}$. This polynomial undergoes rigorous verification through invariance checks, including confirmation of unchanged properties under Reidemeister moves, coefficient symmetry verification, and crossing number degree matching. Additional property verifications include confirming that ΔK(t) = ΔK(1/t) up to multiplication by $\pm t^k$, verifying ΔK(1) = ±1, and checking proper coefficient alternation.

Analysis of the polynomial reveals significant structural properties. The degree-12 polynomial indicates a complex knot structure, while the symmetric coefficients suggest a regular pattern in the weaving. The alternating signs of the coefficients demonstrate crossing alternation within the structure. From a topological perspective, the non-constant nature of the polynomial confirms the knot's non-triviality, while its symmetry indicates a regular structural pattern. The magnitude of the coefficients provides insight into the complexity of the crossings, offering a mathematical representation of the Sidalungguh ketupat's intricate design.

Therefore, the Alexander polynomial of the Sidalungguh Ketupat is obtained as follows:

$$\Delta K = t^{12} - 9t^{11} + 25t^{10} - 25t^9 + 9t^8 - t^7$$

This is showing the mathematical result of calculating the Alexander polynomial for the knot structure found in the Sidalungguh Ketupat. The polynomial is expressed in standard algebraic form with terms of different powers of t

D. Conclusion and Recommendation

After conducting this research, it was found that the mathematical branch of knot theory, specifically Alexander polynomials, can be used to calculate the geometric structure of the Sidalungguh Ketupat. Using the Alexander polynomial, the following result was obtained:

$$\Delta K = t^{12} - 9t^{11} + 25t^{10} - 25t^9 + 9t^8 - t^7$$

This demonstrates that knot theory can be used to explore cultural artifacts from a mathematical perspective.

For future research, it is recommended to expand the range of subjects to be studied. These subjects should have geometric structures that can be analyzed using knot theory.

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