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KNOT THEORY AND CULTURE: AN EXPLORATION OF ALEXANDER POLYNOMIAL IN KETUPAT BATA

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Abstract. The Ketupat bata is an intriguing geometric shape in knot theory, and this study investigates the polynomial knot that was found in the Ketupat . The objective of this study is to analyze the distinctive mathematical aspects of the diamond brick knot polynomial, which include the saturation level and intricate indexing of the polynomial. An examination of the geometric structure of the brick diamond is accomplished through the application of specialist procedures as part of the strategy that is utilized. The knot polynomial that was obtained is found to be a tetrahedral lattice. There is a mathematical relationship that can be represented by the equation

$$A_k = -t^2 + 5t - 8 + 5t^{-1} - t^2.$$

The outcome of this investigation demonstrates how essential mathematical concepts such as its geometric and structural properties.It is possible to produce a more profound comprehension of both culture and nodes by applying the notions of polynomials and nodes to the study of culture.

Keywords: **Knot theory, topology, Alexander polynomial, ketupat Bata, Ethnomathematics**

A. Introduction

The topic of knot invariants is closely linked to the concept of knot polynomials, which has been extensively studied in several scholarly publications (Kadison, 1996). The Jones, Alexander, HOMFLY, and Kauffman polynomials play a crucial role in knot theory as they allow for the distinction between various knots and links. Moreover, these polynomials have a strong connection to the Vassiliev invariants, which are a set of invariants that may classify all knots (Park & Jeong, 2001). It is worth mentioning that knot polynomials have resemblances to other mathematical entities. Sleptsov et al. (2018) have investigated the connection between knot polynomials and the KP hierarchy, showcasing the integrability of these polynomials.

The volume conjecture suggests a connection between the rate of increase of evaluations of colored Jones polynomials and the volume of three-manifolds obtained by performing Dehn surgery along the knot (Sleptsov et al., 2018). (Jackson & Moffatt, 2019) have examined the long-term behavior of the colored HOMFLY polynomial using Chern-Simons invariants and twisted Reidemeister torsion.

Knot polynomials are essential in knot theory, where research papers analyze their properties, relationships with other mathematical ideas, and their uses in knot classification. Several studies provide a thorough examination of the present comprehension and uses of knot polynomials in the area.

The research field investigating the correlation between knot theory and cultural traditions is growing, however to a limited extent (Chen & Ja'Faruddin, 2021; Faiziyah et al., 2020; Hendriyanto et al., 2023; Hidayati & Prahmana, 2022; Ja'faruddin & Chen, 2023; Ja'faruddin & Naufal, 2023; Michael Angrosino, 2557; Muhtadi et al., 2017; Ningsih et al., 2020; Rosa & Orey, 2011; Rosa et al., n.d.; Suharta et al., 2017; Supiyati et al., 2019; Umbara et al., 2023). Ja'faruddin and Haw (2024) conducted a pioneering study in which they utilized knot theory to examine the knot diagram known as "Ketupat Nabi" from the Bugis community in South Sulawesi, as it relates to a traditional culinary practice. This study deepens our comprehension of the mathematical characteristics of cultural artifacts by constructing a distinct Alexander polynomial for the knot diagram associated with this customary supper (Ja'faruddin & Haw, 2024).

Conversely, Goyal (2016) emphasized the importance of cultural competence in nursing practice, specifically for childbearing women from various backgrounds, such as the Asian Indian population. While this study emphasizes the significance of comprehending cultural traditions, it does not have a direct correlation with knot theory or knot polynomials (Goyal, 2016). It is noteworthy that Ja'faruddin and Haw (2024) specifically connect knot theory to cultural heritage, whilst the remaining studies concentrate on the mathematical and theoretical facets of knot polynomials and their utilization in various disciplines, such as physics and theoretical computer science.

While these papers do not directly focus on cultural traditions, they do enhance our understanding of knot polynomials and their significance in mathematical research. While cultural traditions are not directly addressed in these studies, they play a vital role in enhancing our comprehension of knot polynomials and their mathematical applications, particularly when examining the knot diagrams of traditional instruments or meals. Through an analysis of the cultural importance of knot polynomials in traditional rituals and artifacts, these studies enhance our comprehension of the mathematical concepts that form the basis of these customs.

This work aims to investigate the Alexander polynomial of Ketupat Bata. This paper aims to examine the Alexander polynomial of ketupat Bata and its consequences in the field of tropical geometry.

B. Methodolgy

The objective of this research project is to carry out a quantitative analysis using a purposive case study approach to achieve accuracy and a comprehensive understanding of individual instances. This paper aims to examine knot polynomials, particularly the Alexander polynomial, within the framework of Ketupat Bata.

The study utilizes a systematic series of stages, commencing with the choice of a sample of Ketupat Bata and the acquisition of data through meticulous photography and the recording of pertinent information. Geometric mapping and knot tying are accomplished through eye examination, followed by the utilization of MATLAB software to analyze knot polynomials, such as the Alexander polynomial. The results are then interpreted to gain a deeper understanding of how the geometric properties impact the mathematical representation of closed loops.

The study primarily examines the cultural and symbolic significance of Ketupat Bata in traditional civilizations of Southeast Asia. It is a variation of ketupat, a traditional Indonesian meal consisting of rice and coconut milk that is wrapped in coconut or pandan leaves.

The study commences with choosing a sample of ketupat Bata, emphasizing the importance of nodes or links as fundamental elements of the geometric structure. The primary subject is ascertained by capturing photographs of the ketupat and collecting information regarding its philosophy, history, and cultural significance.

The subsequent procedure entails visually examining the knots or connections inside the ketupat and comparing them to existing scientific literature on knot theory. Once the crossings have been identified, a knot diagram is constructed using a Punnett square. This diagram serves as a visual representation and enables a thorough examination of the structure.

Mathematical calculations are conducted for every knot depicted in the diagram, with particular focus on the Alexander polynomial. Understanding the mathematical characteristics of the knots relies heavily on this approach. The research centers on the topological structure and uncovers distinctive attributes necessary for categorization and larger recognition.The study of knot topology involved the exploration of the idea of handedness, also known as chirality, in order to distinguish between left- and right-hand crossings. Handedness is a critical factor in discovering distinct intersections in this research topic.

In the subsequent phase of the investigation, a thorough analysis of the computed polynomial was carried out to reveal the distinctive features of the intersections within the Punnett square. This procedure unveiled the intricacies and practical importance of these intersections.

In order to ensure the accuracy and reliability of the results, a thorough examination of existing literature was conducted. This enhanced the credibility and dependability of the research. In addition, the researchers utilized visual observation, comparative analysis, and mathematical calculations to obtain a thorough comprehension of the topological and cultural significance of ketupat Bata. A comprehensive examination of the pertinent literature was carried out to ensure the precision of the findings.

C. Result and discussion.

Ketupat, or kupat, is a customary Indonesian cuisine consisting of rice enclosed in fresh coconut or palm leaves. Nevertheless, the nomenclature and different manifestations of ketupat can vary across diverse geographical areas. Ketupat bata, a well-known kind of ketupat in Indonesia, possesses a significant cultural heritage intricately linked to the Javanese populace. The precise origin of ketupat bata cannot be definitively verified, however it is thought to have been in existence since the Java kingdoms (Hotima & Hariastuti, 2021).

Figure 1. Ketupat bata

The squares, known as "Ketupat Bata", possibly draw inspiration from the coarse texture of clay bricks commonly employed in construction. Ketupat Bata is not just a culinary delicacy, but also carries significant symbolic significance in Javanese culture. The solid square shape of Ketupat Bata symbolizes robustness, resilience, flawlessness, and equilibrium. Within social settings, it additionally represents fidelity, companionship, and solidarity. The significance of batik in Javanese and Indonesian culture is emphasized through its inclusion in many traditional occasions, such as Idul Fitri festivities, weddings, and other religious ceremonies(Ja’faruddin & Chen, 2023; Ja’faruddin & Haw, 2024). Despite the absence of documented historical records, Ketupat Bata is an essential component of Indonesia's cultural legacy, symbolizing the esteemed principles of harmony, loyalty, and equilibrium cherished by the society.

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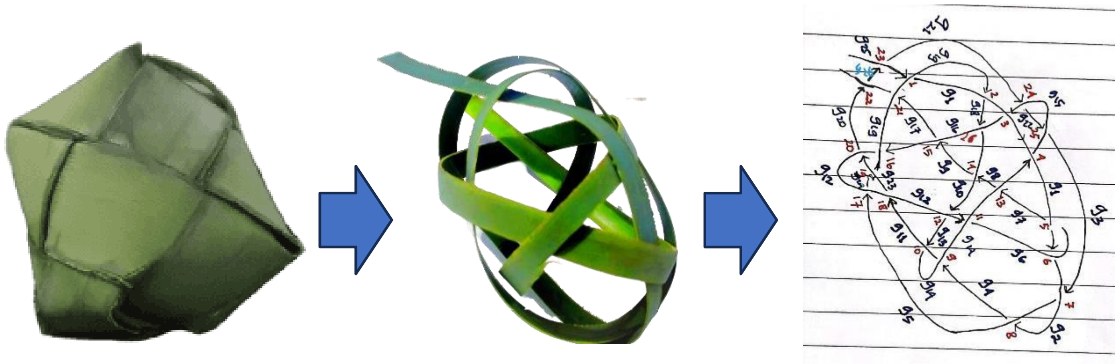

Figure 2. Illustrates the procedure for obtaining a knot diagram.

The diagram presented is in the form of a Ketupat Bata knot, with each junction point assigned a label ranging from 1 to 26, and each line assigned a label ranging from g_1 to g_{26}. The diagram of the Ketupat Bata knot has a total of 26 labels for the intersection locations and 26 labels for the lines.

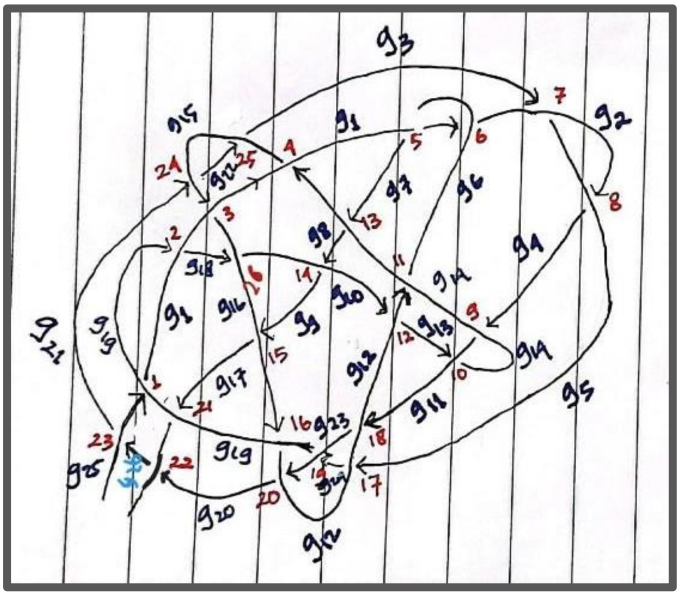

Figure 3. A knot diagram representing Ketupat Bata.

In order to compute the Ketupat Bata polynomial curve, the initial procedure involves employing the right-hand and left-hand rules to determine the lines that are directly linked to the point of intersection. The findings are subsequently displayed in the subsequent table:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
G1	-1	1-t	1-t	1-t	1-t	t	0	0	0	0		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
G2	0	0	0	0	0	-1	1-t	t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
G3	0	0	0	0	0	0	t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0
G4	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
G5	0	0	0	0	0	0	-1	1-t	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0
G6	0	0	0	0	t	1-t	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
G7	0	0	0	0	-1	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
G8	0	0	0	0	0	0	0	0	0	0	0	0	t	t	0	0	0	0	0	0	0	0	0	0	0	0
G9	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	t	0	0	0	0	0	0	0	0	0	0	0
G10	0	0	0	0	0	0	0	0	0	0	0	t	0	1-t	0	0	0	0	0	0	0	0	0	0	0	t
G11	0	0	0	0	0	0	0	0	t	1-t	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0
G12	0	0	0	0	0	0	0	0	0	0	t	1-t	0	0	0	t	1-t	1-t	0	1-t	0	0	0	0	0	0
G13	0	0	0	0	0	0	0	0	0	-1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
G14	0	0	0	-1	0	0	0	0	1-t	t	1-t	0	1-t	0	0	0	0	0	0	0	0	0	0	0	0	0
G15	0	0	t	t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1-t	1-t	0
G16	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	1-t	-1	0	0	0	0	0	0	0	0	0	1-t
G17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	-1	0	0	0	0	0
G18	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1
G19	1-t	t	0	0	0	0	0	0	0	0	0	0	0	0	0	1-t	0	0	-1	0	1-t	0	0	0	0	0
G20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	t	0	0	0	0
G21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	t	-1	0	0
G22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	t	t	0
G23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	t	t	1-t	t	0	0	0	0	0	0
G24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	t	0	0	0	0	0	0	0
G25	t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1-t	1-t	0	0	0
G26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0

Figure 4. The coefficient of Ketupat Bata's Knot Diagram.

In order to discover the polynomial of ketupat bata, computations were performed using MATLAB software to obtain the determinant of the matrix. The final output is as follows.

```
function det_value = calculate_determinant(t)
    syms t;
    A = [
-1  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   (1-t)   0   0   0   0   0   t   0;
(1-t)   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   -1  t   0   0   0   0   0   0   0;
(1-t)   0   0   0   0   0   0   0   0   0   0   0   0   0   t   -1  0   0   0   0   0   0   0   0   0   0;
(1-t)   0   0   0   0   0   0   0   0   0   0   0   0   -1  t   0   0   0   0   0   0   0   0   0   0   0;
(1-t)   0   0   0   0   t   -1  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0;
t   -1  0   0   0   (1-t)   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0;
0   (1-t)   t   0   -1  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0;
0   t   0   -1  (1-t)   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0;
0   0   0   -1  0   0   0   0   0   0   t   0   0   (1-t)   0   0   0   0   0   0   0   0   0   0   0   0;
0   0   0   0   0   0   0   0   0   0   (1-t)   0   -1  t   0   0   0   0   0   0   0   0   0   0   0   0;
0   0   0   0   0   -1  0   0   0   0   0   t   0   (1-t)   0   0   0   0   0   0   0   0   0   0   0   0;
0   0   0   0   0   0   0   0   0   t   0   (1-t)   -1  0   0   0   0   0   0   0   0   0   0   0   0   0;
0   0   0   0   0   0   -1  t   0   0   0   0   0   (1-t)   0   0   0   0   0   0   0   0   0   0   0   0;
0   0   0   0   0   0   0   t   -1  (1-t)   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0;
0   0   0   0   0   0   0   0   t   0   0   0   0   0   0   (1-t)   -1  0   0   0   0   0   0   0   0   0;
0   0   0   0   0   0   0   0   0   0   0   t   0   0   0   -1  0   0   (1-t)   0   0   0   0   0   0   0;
0   0   0   0   -1  0   0   0   0   0   0   (1-t)   0   0   0   0   0   0   0   0   0   0   t   0   0   0;
0   0   0   0   0   0   0   0   0   0   -1  (1-t)   0   0   0   0   0   0   0   0   0   0   t   0   0   0;
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   -1  0   0   0   (1-t)   t   0   0;
0   0   0   0   0   0   0   0   0   0   0   (1-t)   0   0   0   0   0   0   0   -1  0   0   t   0   0   0;
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   -1  0   (1-t)   0   0   0   0   0   0   0;
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   t   0   0   0   0   (1-t)   -1;
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   t   0   0   0   (1-t)   -1;
0   0   0   0   0   0   0   0   0   0   0   0   0   0   (1-t)   0   0   0   0   0   -1  t   0   0   0   0;
0   0   -1  0   0   0   0   0   0   0   0   0   0   0   (1-t)   0   0   0   0   0   0   t   0   0   0   0;
0   0   0   0   0   0   0   0   0   t   0   0   0   0   0   (1-t)   0   -1  0   0   0   0   0   0   0   0];

    det_value = det(A);
end
```

Figure 6. displays the matrix coefficients of Ketupat Bata.

According to the MATLAB output, the quartic polynomial of the Ketupat Bata is obtained as follows

$$A_k = -t^{15} + 5t^{14} - 8t^{13} + 5t^{12} - t^{11}$$

$$A_k = t^{13}(-t^2 + 5t^1 - 8 + 5t^{-1} - t^{-2})$$

$$\approx -t^2 + 5t^1 - 8 + 5 * t^{-1} - t^{-2})$$

$$A_k = -t^2 + 5t^1 - 8 + 5 * t^{-1} - t^{-2})$$

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D. Conclution and suggestion

In this research endeavor, the theoretical framework of knots (knot theory) was effectively integrated into the geometric configuration of Ketupat Bata. By calculating the Alexander polynomial of the Ketupat Bata, profound insights were attained into the intricate and mathematical relationships that exist within traditional culture, as well as the robustness of the social bonds that unite society. The polynomial knot derived from the Ketupat Bata is as follows:

$$A_k = -t^2 + 5t^1 - 8 + 5 * t^{-1} - t^{-2})$$

The study's findings elucidate the mathematical aspects of knots and links. In this case, the form and pattern of ketupat Bata as a cultural symbol can be interpreted as a knot in mathematics. When applying Alexander's polynomial theory to model, we can generate polynomials that illustrate its geometric and structural characteristics. This demonstrates how fundamental mathematical concepts, such as knots and polynomials, can be applied to culture to foster a deeper comprehension of both. Hence, the incorporation of mathematics in this context not only enhances our understanding of ketupat as a cultural symbol but also enriches our perspective on the interconnection between mathematics and culture generally.

Future research should delve deeper into the applications of Alexander's polynomial simplices theory in diverse cultural symbols by adopting a multidisciplinary approach. This study can explore the connections between the geometric forms of cultural symbols and concepts in the theory of simplices and apply pertinent mathematical methods for a more comprehensive analysis.perspective on the relationship between mathematics and culture as a whole.

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