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COVARIATIONAL REASONING PROFILE OF PROSPECTIVE MATHEMATICS TEACHER STUDENTS WITH *FIELD-INDEPENDENT* COGNITIVE STYLE IN SOLVING COVARIATION PROBLEMS

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Abstract. Covariational reasoning is closely related to the problem of the relationship between two variables which involves visual abilities in constructing graphs. A visualization is an essential tool in solving math problems. Measurement of visual ability can be seen from the *field-independent* and *field-dependent* cognitive styles. This study aims to describe the covariational reasoning of prospective mathematics teacher students with a *field-independent* cognitive style in solving covariation problems. This type of research is qualitative research with the research subject of prospective mathematics teacher students at STKIP PGRI Jombang selected based on the GEFT test by controlling for gender equality and math ability. The subject obtained is a student with a *field-independent* cognitive style with a GEFT score. The instruments used in this study were the main instrument (researchers themselves) and supporting instruments in the form of GEFT, TKM, TK, and interview guides. The research data obtained were analyzed through a process of data reduction, data presentation, and conclusion. The results of the study show that prospective mathematics teacher students with *field-independent* cognitive styles identify covariation problems by determining the variables contained in the covariation problems and constructing relationships between the two variables that have been determined; coordinating the magnitude of variable changes and determining the changing pattern by dividing the bottle into several parts to arrange the changing pattern; construct through the representation of the relationship of two variables into a graph by determining the coordinate axes with the known variables and drawing a graph of the pattern of changes in the relationship of the two variables.

Keywords: Covariational Reasoning, *Field-Independent*

A. Introduction

Reasoning is the foundation of learning mathematics. Reasoning is part of a person's thought process. According to NCTM (2002:56), mathematical thinking is the habit of thinking and its use in many contexts can consistently establish reasoning. The reasoning is a part of the thought process that has certain characteristics that are logical thought processes or thought processes that are analytical. The characteristics of reasoning are (1) the existence of a mindset called logic. In this case, it can be said that reasoning activities are a logical thought process. This logical thinking is defined as thinking according to a certain pattern or according to a certain logic; (2) the thought process is analytical. One of the goals in the process of learning mathematics is the ability to reason and solve problems. Understanding problems and reasoning is a cognitive component that students must have in the problem-solving process. One of the mathematical concepts that require reasoning in understanding is the concept of function. Weaknesses in understanding the concept of function cause students to have difficulty in the process of solving problems related to concepts such as the problem of dynamic events. The discussion of the concept of functions is related to the discussion of graphs. Graphics involve interpretation and construction. Interpretation always points to the student's ability to read the graph and capture information from the graph.

Understanding the concept of functions closely related to graphs became the basis for the understanding of the concept of graphs in calculus. Carlson et al (2002) state that students are still lacking in terms of the ability to interpret graphs. Students have difficulty representing and interpreting changes in function graphs. In line with Koklu's research (2007) which states that students tend to think procedurally about changes in function to hinder reasoning ability. According to Thompson&Carlson (2017), the covariation approach shows a "relationship" between two quantities expressed algebraically, visually in graphs, or in real-world situations. Analyzing, coordinating, and understanding as a whole between changes in quantity describes covariation activities (Slavit, 1997). Carlson et al (2002) define the cognitive activity that involves the coordination of two kinds of quantities by paying attention to the change in the quantity of one against the other referred to as covariational reasoning.

Covariational reasoning initially emerged as a theory expressed by Confrey in the late 1980s and by Thompson in the late 1990s. The value of consecutive variables becomes the focus of the covariational reasoning put forward by Confrey. Meanwhile, the measurement of traits in objects is the focus of covariational reasoning according to Thompson. Both describe coordination as the basis for covariational reasoning regarding the relationship of dynamic events. Confrey describes covariational reasoning as a discrete approach that focuses on changing the value of variables. Saldanha&Thompson (1998) describe continuous covariational reasoning with a focus on measuring the properties of objects and simultaneous changes continuously. Carlson *et al* (2002) define covariational reasoning as a cognitive activity that involves the coordination of two kinds of quantities related to how the two quantities change one against the other. Carlson *et al* (2002) developed a covariation framework that describes five mental actions and five levels of covariational reasoning. The development of the aspects to be studied is summarized in the reasoning indicators presented in the following table:

Table 1. Covariational Reasoning Indicators

Aspects of covariational reasoning according to researchers	**Indicator**
Identify	Determining the corresponding variables of the covariation problem Qualitatively explain the relationship between two variables
Coordinating	Determining the pattern of change between the two variables and determining their direction Determining the magnitude of the change of one variable when viewed from another variable
Constructing	Represents the relationship between variables in general in a graph

Covariational reasoning can be developed through dynamic event modeling. Students need to be faced with problems related to the concept of dynamic events that involve the relationship between two variables. A dynamic event is an event that describes a change in the value of a variable that causes a change in the value of another variable. The problem of covariation is defined as the assignment of covariation relating to dynamic events involving a relationship between two variables. Covariation problems include problems involving the relationship of two variables such as time and position, time and distance, volume and height, residual volume of water and time, interpretation of graphs, and so on involving the coordination of two variables.Individuals who have a *field-independent* cognitive style are more critical, they can choose a stimulus based on the situation. Witkin said that individuals who have a *field-independent* cognitive style prefer to separate parts of a number of patterns and analyze patterns based on their components. Individuals who have a cognitive style *field-independent* have more analytical characteristics where the individual is less dependent or less influenced by the environment.

Covariational reasoning involves the visual ability to construct graphs on covariation problems. When understanding information covariation problems, a person with a *field-independent* cognitive style can separate the information used even though the information is in a broader context whereas the dominant environment affects one's work *field-dependent.* The measurement of spatial visual ability can be seen from the cognitive styles of *field-independent* and *field-dependent* (An&Car, 2017). The ability to measure spatial visuals is needed in constructing graphs which is one aspect of covariational reasoning.

The identification of covariational reasoning in *field-independent* and *field-dependent* cognitive styles involves visual abilities when building graphs on covariation tasks. A visualization is an important tool in solving mathematical problems. It is therefore quite possible that individuals with different cognitive styles also show covariational reasoning at different levels. This study aims to describe covariational reasoning in solving covariation problems in prospective teacher students with *field-independent* cognitive styles.

B. Research Methods

The type of research used in this study is qualitative research. Researchers use this type of research because it is relevant to the purpose of the study, which is to describe the covariational reasoning profile of prospective mathematics teacher students with *a field-independent* cognitive style in solving the problem of covariation.

The subjects in this study were students of the Mathematics Education Study Program Class of 2017 who were selected based on the GEFT *(Group Embedded Figures Test)* test and selected a student with *a field-independent* cognitive style with a score of 15 and high mathematical ability with a score of 85 which is further encoded with SFI. The instruments in this study consist of the main instruments, namely the researcher himself, and supporting instruments in the form of GEFT, Mathematical Ability Test (TKM), TK (Covariation Task), and interview guidelines. The percentage of *field-dependent* and *field-independent* subjects in students of the Mathematics Education Departmen Class of 2017 based on the GEFT test is presented in the following figure:

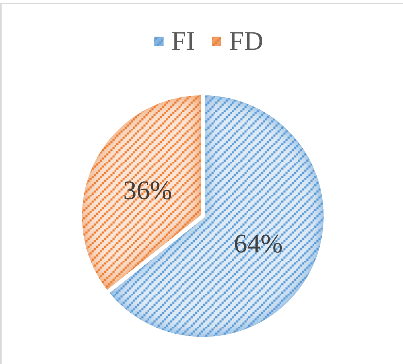

Figure 1. GEFT Results

In this study, researchers used time triangulation techniques to ensure the validity of the data. TKM and TK instruments are validated by Mathematics Education Lecturers.

Table 2. Covariation Tasks

If the shape of the bottle is like the picture on the side, draw a graph showing the height of the water as a function of the amount of water that is constantly filled into the bottle. Give your reasons!	

Data collection was carried out with task-based interviews. The subject did the TK questions by writing the answers on the answer sheet and conducting interviews with the research subjects to check and clarify the results of the completion of the research subjects. Furthermore, the valid data is analyzed. Data analysis includes: (1) data reduction, (2) data presentation, (3) conclusions.

C. Results and Discussion

Based on research data, the results of the study were obtained as follows:

1. Identifying aspects

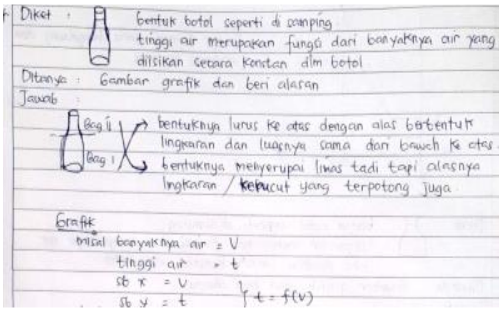

Figure 2. SFI's answers on identifying aspects

The results of the interview obtained from SFI related to the identifying aspect are as follows:

Table 3. SFI interview results on identifying aspects

P	What information do you get from the question?
SFI	Here there is a bottle, then the height of the ice is a function of the amount of water that is filled constantly in the bottle.
P	How does the bottle look?
SFI	In my opinion, the shape of this bottle is a circle divided into 2 parts.
P	How do you solve it?
SFI	The bottle consists of 2 parts. Part 1 is straight up with a circular surface and in part 2 the shape is getting smaller so that the surface is getting smaller.
P	Then what steps do you use?
SFI	First, I suppose the x-axis is the amount of water equal to V and the y-axis that is the height of the water, next I want to draw $t = f(V)$.

SFI writes down information based on the questions provided. The SFI divides the known bottle into several parts and is captioned next to it as a form of representation of the subject in identifying the problem. SFI determines these variables based on the problem, namely x as the amount of water that is further annotated V by the volume and y as the level of water in the bottle which is annotated with t

2. Aspects of coordinating

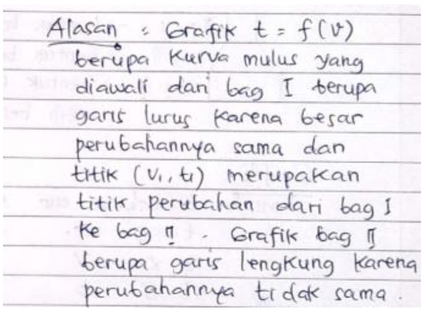

Figure 3. SFI's answer on the aspect of coordinating

The results of the interview obtained from SFI related to the aspect of coordinating are as follows:

Table 4. SFI interview results on the aspect of coordinating

P	How to draw the graph?
SFI	Now there is part 1 because the surface area is the same then the graph is in the form of a straight line then part 2 because the shape is getting smaller upwards, the graph is in the form of a curved line so in my opinion when it is depicted in the form of a smooth curve that starts from part 1 in the form of a straight line and V_1, t_1 is a point of change from part 1 to part 2.
P	What caused the chart in part 2 to be a curved line?
SFI	Yes, because the shape of the bottle was not the same surface area so that the more filled with water, the change in height was not the same.
P	How is the relationship between the two variables on this issue?
SFI	If the relationship seems to me to be that the bottle is straight up, if water is added constantly, the change in height also remains as the number 1 question was. For those whose bottle narrows upwards, when water is added constantly, the change in height is getting bigger and bigger.

Based on the parts of the bottle made, SFI formulates a height change of each part of the bottle which will then be used in constructing the graph. SFI explains that when the bottle is straight the height change will remain and for bottles that are tilted in shape, the change in height is not the same. This will affect the graphic shape of the bottle.

3. Constructing aspects

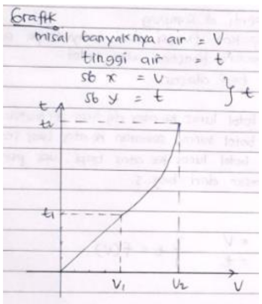

Figure 4. SFI's answer to the constructing aspect

The results of the interview obtained from SFI related to the constructing aspect are as follows:

Table 5. SFI interview results on the aspect of constructing

P	Then what steps do you use to draw the chart?
SFI	Now there is part 1 because the surface area is the same then the graph is in the form of a straight line then part 2 because the shape is getting smaller upwards, the graph is in the form of a curved line so in my opinion when it is depicted in the form of a smooth curve that starts from part 1 in the form of a straight line and V_1, t_1 is a point of change from part 1 to part 2.
P	What caused the chart in part 2 to be a curved line?
SFI	Yes, because the shape of the bottle was not the same surface area so the more filled with water, the change in height was not the same.
P	What is the relationship between the two variables on this issue?
SFI	First I suppose the x-axis is the amount of water equal to V and $y = t$ that is the height of the water, next I want to draw $t = f(V)$
P	How to draw the graph?
SFI	Now there is part 1 because the surface area is the same then the graph is in the form of a straight line then part 2 because the shape is getting smaller upwards, the graph is in the form of a curved line so in my opinion when it is depicted in the form of a smooth curve that starts from part 1 in the form of a straight line and V_1, t_1 is a point of change from part 1 to part 2
P	So what's the direction?
SFI	If the direction of the chart is upward.

P	Why?
SFI	Because basically on the bottle when added the high automatic volume of water in the bottle will continue to grow and that's what causes the upward direction of the chart.
P	What will happen to the graph you create if I add water when the bottle is full?
SFI	Yes, the height of the water in the bottle remains the same as the height of the bottle, ma'am.
P	So what does the graph look like?
SFI	When the bottle is full, the graphic shape is in the form of a horizontal straight line.

SFI represents the relationship of the specified variable V and t through a graphic image constructed through the parts that the SFI makes on the shape of the bottle or jug. The difference in the shape of the graph is influenced by the shape of the bottle, causing a large change in height that is not fixed. The direction of the graph created is always pointing upwards because every time water is added constantly, the height will also increase. SFI also explains that when the bottle is full and water is added again, the graph will be a horizontal straight line because the height of the water after it is full will remain the same as the height of the bottle.

Based on the results of this study, there is a discussion of the results on several aspects of covariational reasoning. In the **identifying aspect**, *field-independent* subject determines the corresponding variables of the covariation problem as well as compiles the relationship between the two variables summarized in each reason for the given problem. This is in line with an opinion (Saldanha&Thompson, 1998; Madison et al., 2012; Kertil, 2020) which states that the concept of covariation begins with identifying two data sets. In the **aspect of coordinating**, *field-independent* subject determines the magnitude of the change in water height to the volume of water given or vice versa through an understanding of the shape of the bottle. After knowing the magnitude of the change of the specified variable, the subject determines the pattern of change in water height see if the relationship between the two variables has increased, decreased, or even fixed. This is in line with an opinion (Confrey&Smith, 1994; Slavit, 1997; Saldanha&Thompson, 1998; Carlson et al, 2002) which involves the coordination aspect of two-variable change in covariation reasoning. The results of this study on the aspect of coordinating show that the subject has not established the multiplication of objects based on known variables as a result of uniting the measured properties of simultaneously changing quantities This is contrary to the definition of covariational reasoning according to Thompson&Carlson (2017) states that the level of covariational reasoning includes coordination involving the multiplication of objects. On the **constructing aspect**, the subject can present the relationship of variables on the problem of covariation into a graph. In line with an opinion (Johnson et al., 2017; Kertil, 2020) which involves the constructing aspect of covariational reasoning is constructing variable changes in an image and forming a simultaneous image of the covariance properties of two variables. The subject begins the process of constructing a graph, namely providing the coordinate axis with variables that have been labeled from the covariation problem, determining the points, and connecting the points into a dynamic function graph. When associated with cognitive styles in constructing function graphs, visual abilities are needed when building graphs on covariation problems (An&Car, 2017).

The novelty in this study lies in the aspect of covariational reasoning which is a slice of the reasoning aspect put forward by several experts who discuss covariational reasoning. Although the core definition of covariational reasoning is more in the aspects of coordination and construction, this study also adds aspects of covariational reasoning from experts, namely the identifying aspect developed in covariational reasoning indicators. This study does not describe the subject at the level of covariational reasoning but rather emphasizes the student's thought process that is adapted to aspects of covariational reasoning. The instruments used by researchers are still limited to positive covariation, so it is still necessary to conduct research that includes negative and zero covariation problems such as dynamic events regarding changes in speed and time.

D. Conclusion

Based on the results of the study, it was concluded that in the identifying aspect, *field-independent* subjects identified variables related to the problem of covariation through the representation of known information on the question. The *field-independent* subject identifies the information of the problem by creating parts on the bottle that are known to be the basis of the *field-independent* subject in determining the magnitude of the change in the amount of water or the volume and height of the water in the bottle. In the aspect of coordinating, *field-independent* subject determines the pattern of change between the two variables that have been identified. How variables change with the increment of variables. The *field-independent* subject makes parts of the bottle that the *field-independent* subject thinks are irregular which causes a large change in height and is also not the same. However, in all bottles, when water is added, the height will increase, causing the direction of the chart to go up. The magnitude of the change in the variable results in the direction created by *field-independent* subject. In the constructing aspect, *field-independent* subject represents the relationship between the two variables defined in a graph. The *field-independent* subject is constructed based on a bottle divided into parts first to formulate the magnitude of the change in height. So the shape of the bottle affects the construction of graphic drawings. In constructing a graph, the *field-independent* subject also specifies the coordinate axis as a predetermined variable before drawing the graph.

For all aspects of covariational reasoning to develop well in students, it is recommended for lecturers/teachers to provide tasks that can measure all these aspects in the form of a covariation problem. Furthermore, in understanding the problem of covariation related to mental constructions, this covariational reasoning is related to the theory of APOS, thus it is advisable for subsequent researchers to look at mental constructions on covariational reasoning using APOS theory.

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