GEOMETRIC CONCEPTS IN JAMBI BATIK: DEVELOPING A HYPOTHETICAL LEARNING TRAJECTORY FOR REFLECTION MATERIAL

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Abstract

Learning geometric transformations often presents challenges for students due to their abstract nature and limited connection to real-world contexts. Reflection is one of the most difficult concepts for students to understand. This study aims to design a Hypothetical Learning Trajectory (HLT) for geometric reflection using the PMRI approach through the context of Batik Jambi. This research employed a design research approach in the preliminary design phase, with data collected through teacher interviews and literature studies. The result is a five-stage HLT design that utilizes the Bungo Melati motif as a symmetry context, guiding students from concrete experiences toward formal understanding through progressive mathematization. The findings indicate that incorporating the Batik Jambi context can enhance students' comprehension of reflection while also connecting mathematics learning with local cultural values. This HLT design has the potential to serve as a foundation for developing a Local Instructional Theory (LIT) for more meaningful and contextualized geometry learning.

Keywords: Hypothetical Learning Trajectory, Geometric Reflection, PMRI, Batik.

A. Introduction

Reflection is one of the concepts in geometric transformation that most frequently presents difficulties for students because it requires the ability to understand how the position of an object changes relative to a given axis. Many students memorize coordinate transformation rules without truly understanding how points move when reflected, leading to common errors in determining the image of an object or mapping points on the coordinate plane (Yuliardi & Rosjanuardi, 2021). This situation is exacerbated by learning processes that insufficiently emphasize exploratory and contextual activities, making it difficult for students to relate the concept of reflection to real-world phenomena (Andelia et al., 2022). Limited spatial visualization skills also hinder students from connecting the reflection process with geometric forms they observe. This condition indicates that learning reflection requires visual representations and exploratory activities that support students in observing positional changes concretely, as emphasized by Hlongwana et al, (2025) who argue that the ability to visualize reflection results plays a crucial role in developing meaningful spatial understanding and geometric reasoning.

Cultural contexts that exhibit symmetrical structures can serve as an effective bridge to help students naturally understand the concept of reflection. Jambi Batik motifs, particularly the Bungo Melati pattern, display symmetry along vertical and horizontal axes, enabling students to identify opposite elements of the motif and experience the reflection process through drawing activities (Novrika et al., 2016). This visual experience provides a concrete foundation before students are introduced to coordinate representations and formal reflection rules, allowing concepts to be understood more gradually and meaningfully.

The use of batik as a contextual basis in mathematics instruction is chosen because batik functions not only as an Indonesian cultural heritage but also possesses rich mathematical structures that align closely with the principles of Realistic Mathematics Education in Indonesia (PMRI). Batik motifs such as *kawung*, *parang*, and *melati* contain repetitive patterns, symmetry, transformations, and other geometric structures that enable students to construct mathematical concepts through meaningful contexts. Research has shown that batik serves as an effective medium for bridging students' understanding from informal situations to formal mathematical representations. Marissa et al. (2024), for instance, found that the Kawung Batik motif supports the development of a learning trajectory in arithmetic sequences and series. In the context of geometric transformations, Sahara et al. (2024) demonstrated that batik-based activities, both physical and digital help students comprehend reflection, translation, and rotation more intuitively.

The selection of batik over other cultural objects is further supported by ethnomathematics findings that consistently show that the batik-making process involves genuine mathematical activities such as measurement, estimation, designing symmetric patterns, and calculating proportions (Abdullah et al, 2025). Moreover, several studies affirm that batik motifs can represent complex mathematical concepts such as fractals, tessellations, and transformational geometric patterns (Fachrunnisa & Sari, 2023; Sulistyawati & Rofiki, 2022). Because batik is closely integrated into students' daily lives and provides authentic learning opportunities, this context aligns with PMRI principles, which emphasize reality, knowledge construction, and progressive mathematization. Thus, batik becomes the most relevant, meaningful, and empirically supported choice as a context for mathematics learning.

Various studies have demonstrated that local cultural contexts are effective in helping students grasp the concept of reflection. Maifa et al, (2020) developed a learning trajectory for reflection using the Buna textile context, Maryati & Prahmana (2021) designed learning trajectories for dilation and reflection through bamboo weaving patterns, and Sanita et al, (2024) showed that the Songket Prada Palembang context supports students in understanding reflection progressively. These findings confirm that local cultural contexts can serve as effective starting points for meaningful mathematics learning.

To ensure that context-based learning proceeds systematically and aligns with students' cognitive development, a clear framework is needed in its design, namely the Hypothetical Learning Trajectory (HLT). HLT is a key component of the Learning Trajectory (LT) that describes the anticipated progression of students' thinking throughout the learning process (Gravemeijer & Cobb, 2006). HLT consists of three essential elements: learning goals, learning activities, and hypotheses about students' learning development. According to Gustiningsi et al. (2023), a well-constructed HLT serves as a conceptual guide that enables teachers to predict students' responses and develop activities aligned with their cognitive abilities. Thus, an HLT design functions not only as a tool for instructional planning but also as a reflective instrument for building local instructional theory.

Based on these needs, this study aims to design a Hypothetical Learning Trajectory (HLT) for geometric reflection grounded in the PMRI approach by utilizing the context of Jambi Batik drawing, particularly the *Bungo Melati* motif. This study was conducted during the preliminary design phase of design research, resulting in an HLT design without field implementation. The resulting design is expected to facilitate students' progressive mathematization, strengthen the connection between mathematics and local culture, and serve as a foundation for developing a Local Instructional Theory (LIT) for contextual and meaningful learning of geometric reflection.

B. Methods

This study employed a Design Research methodology with a focus on the preliminary design phase, which involves developing a Hypothetical Learning Trajectory (HLT) without field implementation. According to Gravemeijer dan Cobb (2006), design research aims to develop contextual instructional theory through the processes of design, implementation, and retrospective analysis.

During the preliminary design phase, the research activities centered on conceptual analysis and the development of a hypothetical design grounded in theory, local context, and initial analysis of students' characteristics as well as the learning content of geometric reflection. The research data were obtained through literature review, exploratory interviews with mathematics teachers, and an analysis of the local cultural context of Jambi Batik relevant to the concept of reflection.

The outcome of this phase is a Hypothetical Learning Trajectory (HLT) designed using the Indonesian Realistic Mathematics Education (PMRI) approach, which is ready to be tested in the next stage of the study (teaching experiment). Thus, the study focuses on constructing a hypothetical learning trajectory model that integrates the concept of geometric reflection with the cultural context of painting Jambi Batik.

C. Result and Discussion

The preliminary design phase of this study produced a Hypothetical Learning Trajectory (HLT) for teaching geometric reflection using the context of painting Jambi Batik with the Bungo Melati motif. This design was developed based on the principles of Indonesian Realistic Mathematics Education (PMRI) and the theory of progressive mathematization, which guides students from concrete contexts toward formal representations (Gravemeijer & van Eerde, 2009; Zulkardi & Ilma, 2006).

This HLT consists of five main learning stages, each representing a shift in students' levels of thinking in understanding the concept of reflection. The selection of these five stages was made because each stage illustrates a specific cognitive transition from contextual activities to the formalization of concepts. According to Simon (1995), an effective HLT must include sufficient transitional phases to allow students to construct conceptual bridges between concrete experiences and mathematical symbols.

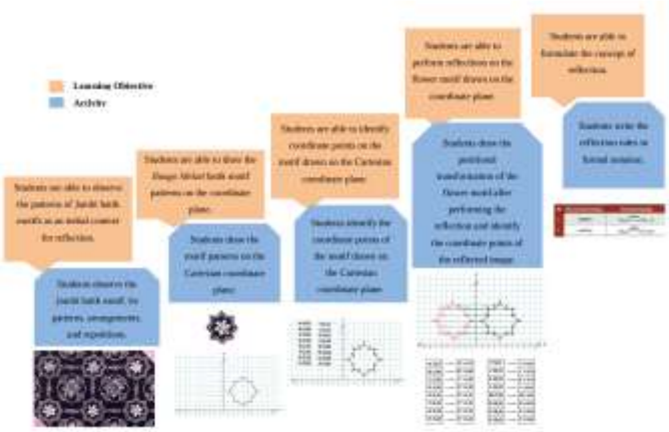

Figure 1. The designed HLT

This diagram illustrates the learning trajectory of geometric reflection through the context of Jambi Batik, starting from observing the *Bungo Melati* motif to discovering the formal rules of reflection on the Cartesian coordinate plane.

Figure 2. The Jambi Batik motif *"Bungo Melati"*

Table 1. The designed HLT for reflection learning

Stage	Learning Objective	Activity	Conjecture of Students' Thinking
Informal	Students are able to observe the patterns of Jambi batik motifs as an initial context for reflection.	Situational analysis by observing the patterns on the *Bungo Melati* batik motif.	Students discuss and observe the patterns on the batik motif.
Model of	Students are able to draw the *Bungo Melati* batik motif	Drawing the observed batik motif patterns.	Students draw the motif patterns and discuss how the patterns shift.

Stage	Learning Objective	Activity	Conjecture of Students' Thinking
	patterns on the coordinate plane.		
Model for	Students are able to identify coordinate points on the motif drawn on the Cartesian coordinate plane.	Identifying coordinate points of the motif drawn on the Cartesian plane.	Students determine pairs of coordinate points.
	Students are able to perform reflections on the flower motif drawn on the coordinate plane.	Drawing the shifted position of the flower motif after performing the reflection and identifying the coordinate points of the reflection result.	Students draw the patterns along with their symmetries on the Cartesian coordinate plane and determine the coordinate points resulting from the reflection.
Formal	Students are able to formulate the concept of reflection.	Writing reflection rules in formal mathematical notation.	Students derive and write formal reflection rules based on exploration and discussion. $P(x,y) \xrightarrow{M_x} P'(x,-y)$ $p(x,y) \xrightarrow{M_y} P'(-x,y)$

The stages of the designed HLT are as follows:

1. **Observing the Bungo Melati Motif Pattern**

   Students observe the *Bungo Melati* motif on batik cloth and identify parts of the pattern that repeat or face each other. This activity aims to help students recognize the presence of symmetry in the batik pattern.

2. **Drawing the Batik Motif Pattern on the Coordinate Plane**

   Students draw one of the motifs onto the coordinate plane in a simplified form. This activity introduces the relationship between visual shapes and the positions of points in the coordinate system.

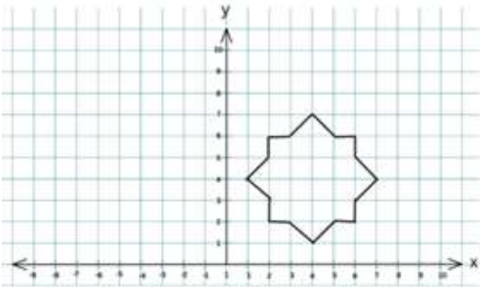

Figure 3. The motif pattern on the coordinate plane

3. **Identifying Coordinate Points on the Motif**

   Students determine the coordinate points of parts of the motif (for example, the tips of the flower petals) and write down the positions of these points to facilitate the subsequent reflection process.

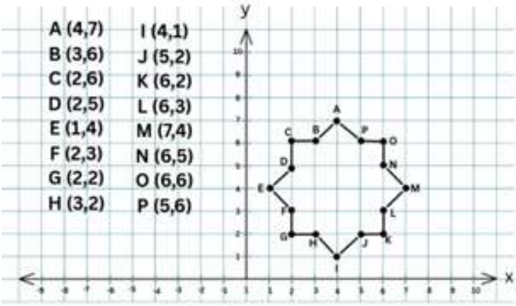

Figure 4. Identification of coordinate points on the pattern

4. **Performing Reflections on the Motif**

   Students reflect the identified points across the x-axis and y-axis. This activity helps them understand how the positions of points change when a reflection is performed.

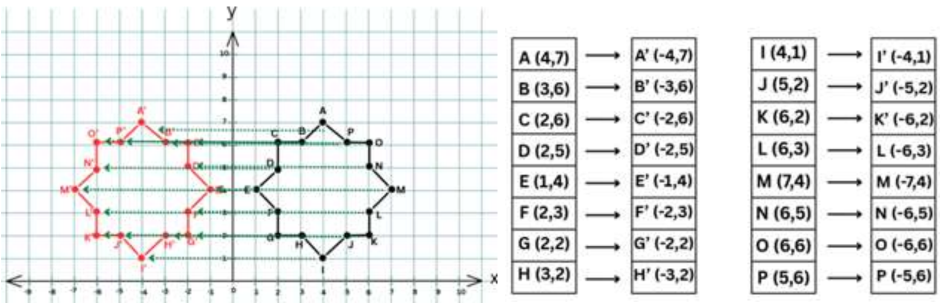

Figure 5. Pattern reflection and point identification

5. **Discovering the Reflection Rules**

   Students compare the positions of the original points and their images to identify the coordinate transformation patterns, such as $(x, y) \rightarrow (x, -y)$ for reflections across the x-axis and $(x, y) \rightarrow (-x, y)$ for the y-axis. At this stage, students begin to understand the formal concept of reflection.

Each stage is designed to enable students to construct their understanding progressively. The *Bungo Melati* pattern is used because it exhibits clear symmetry along both vertical and horizontal axes and allows for a natural transition from artistic activities to formal geometric concepts (Hidayati & Sugeng, 2021).

The design of this HLT is grounded in the theory of Gravemeijer and van

Eerde (2009), which emphasizes that a hypothetical learning trajectory serves as a guide for teachers in predicting students' thinking processes toward conceptual understanding. The selection of five stages in this HLT is deliberate; the structure follows the ideal sequence of progressive mathematization, namely: concrete situations → situational models → mathematical models → formal symbols.

In the first and second stages, students interact directly with the concrete context through activities of observing and drawing the *Bungo Melati* motif. According to Yuliardi dan Rosjanuardi, (2021), this phase is essential because visual representations act as a bridge from real-life activities to the formal concept of reflection. Through motif observation, students begin to identify axes of symmetry and recurring positional shifts, which serve as the foundation for understanding reflections across a line.

The third and fourth stages reinforce the process of horizontal mathematization, in which students begin to connect visual patterns with coordinate representations. In this phase, they mark key points and perform reflections across axes, helping them understand positional changes quantitatively. Consistent with the findings of Salsabila & Hajizah, (2024) activities that focus on visual and coordinate representations can enhance students' spatial abilities and facilitate the transition toward abstract thinking.

The fifth stage marks vertical mathematization, which is the transition from situational models to formal mathematical models. Students begin to derive reflection rules from their own observations. This aligns with the principle of self-construction in PMRI, which emphasizes that mathematical knowledge should be constructed through reflective processes rather than delivered directly (Zulkardi & Ilma, 2018).

The choice of the Jambi batik painting context, particularly the *Bungo Melati* motif, strengthens the connection between cultural activities and mathematical concepts. The *Bungo Melati* pattern exhibits clear lines of symmetry and reflection, making it an ideal context for constructing an understanding of reflection. This context also enables students to experience transformation concepts as meaningful visual and aesthetic activities, as described by Gustiningsi et al. (2023) who note that the success of an HLT design is greatly influenced by the alignment between real-life contexts and the mathematical concepts being learned.

This HLT design also demonstrates that the process of understanding geometric reflection can be developed progressively, from concrete motif observation to the discovery of formal rules. The five-stage structure is considered the most appropriate because each stage acts as a cognitive bridge to the next level of abstraction. If the number of stages were reduced to four, the transition from visual observation to coordinate representation would be omitted, potentially hindering students' ability to understand spatial relationships comprehensively (Yuliardi & Rosjanuardi, 2021). Thus, the five-stage design provides a balanced integration of exploration, representation, and formalization, which is crucial in HLT and PMRI based learning.

D. Conclusion

The preliminary design stage of this study successfully produced a Hypothetical Learning Trajectory (HLT) for geometric reflection using the Indonesian Realistic Mathematics Education (PMRI) approach and the contextual activity of painting Jambi Batik with the *Bungo Melati* motif. This design emphasizes the development of students' mathematical thinking from concrete experiences to formal understanding of reflection through five learning stages: observing the motif, drawing on the coordinate plane, identifying points, performing reflections, and discovering reflection rules.

Each stage functions as a cognitive bridge that guides students through progressive mathematization, from contextual observation to formal reasoning. The use of the *Bungo Melati* motif enables students to connect visual and cultural experiences with the concept of reflection, making the learning process more meaningful and interactive.

Overall, this study demonstrates that integrating the PMRI approach with local cultural contexts can facilitate more meaningful mathematics learning. The resulting HLT design can serve as a foundation for developing a Local Instructional Theory (LIT) for contextual and systematic geometry reflection learning rooted in Jambi's cultural heritage.

Reference

Abdullah, A. S., Richardo, R., & Mubarok, C. (2025). *Ethnomathematics of Batik Giriloyo: Mathematical practices in local cultural production*. Journal of Education, Culture and Society, 16(1), 45–60.

Andelia, I. S. K., Wijayanti, R. . R., & Faulina, R. (2022). Analisis Pemahaman Konsep Geometri Transformasi dalam Penerapan Etnomatematika Budaya Batik Tulis Tanjung Bumi. *Jurnal Cendekia : Jurnal Pendidikan Matematika*, *6*(2), 1797–1809. https://doi.org/10.31004/cendekia.v6i2.1384

Fachrunnisa, R., & Sari, F. K. (2023). Transformasi geometri pada motif Batik Melati Desa Kebon Bayat Klaten. *Jurnal Cendekia: Jurnal Pendidikan Matematika*, 7(1), 112–124.

Gravemeijer, K., & Cobb, P. (2006). Design research from the learning design perspective.

Gravemeijer, K. & Eerde, V. S. (2009). Design research as a Means for Building aKnowledge Base fo Teaching in Mathematics Education. The Elementary School Journal. Volume 109 Number 5.

Gustiningsi, T., Putri, R. I. I., Zulkardi, & Hapizah. (2023). Designing Hypothetical Learning Trajectory for Rectangular Area Using Patchwork. AIP Conference Proceedings, 2811(1), 020009. American Institute of Physics. https://doi.org/10.1063/5.0142279

Gustiningsi, T., & Utari, R. S. (2020). Penerapan Pendekatan PMRI Bagi Relawan Komunitas Pendidikan Dalam Pembelajaran Matematika. Prosiding Seminar Nasional Pendidikan Program Pascasarjana, Universitas PGRI Palembang, Indonesia.

Hidayati, & Sugeng. (2021). Penerapan transformasi geometri pada desain batik Lia Maido menggunakan Desmos. Jurnal PRIMATIKA, 10(2), 1–8.

Hlongwana, P. (2025). *Enhancing geometry problem-solving through visualization for multilingual learners*. *21*(7).

Maifa, T. S., Salsinha, C. N., & Bete, H. (2020). Lintasan Belajar Geometri Transformasi Dengan Menggunakan Konteks Kain Buna. HISTOGRAM: Jurnal Pendidikan Matematika, 4(2), 439–450. https://doi.org/10.31100/histogram.v4i2.696

Marissa, A., Sari, Y. I., Islam, M. S., & Gultom, S. (2024). Local wisdom "batik kawung": Developing learning trajectory of mathematics content in sequence through realistic mathematics education. *Journal of Sustainable Development*, 4(2), 50–62.

Maryati, M., & Prahmana, R. C. I. (2021). Learning Trajectory of Dilation and Reflection in Transformation Geometry through the Motifs of Bamboo Woven. *Jurnal Didaktik Matematika*, *8*(2), 134–147. https://doi.org/10.24815/jdm.v8i2.21283

Novrika, D., Putri, R. I. I., & Hartono, Y. (2016). Desain Pembelajaran materi Refleksi menggunakan motif kain batik untuk siswa kelas VII. *Prosiding Seminar Matematika Dan Pendidikan Matematika*, *November*, 607626. http://jurnal.fkip.uns.ac.id

Sahara, R., Dolk, M., Hendriyanto, R., Kusmayadi, T. A., & Fitriana, T. (2024). Geometry transformation as a part of the design of digital manipulative activity based on batik: A local instructional theory. *Journal of Mathematics Education*, 15(3), 233–250.

Salsabila, E., & Hajizah, M. N. (2024). *Hypothetical Learning Trajectory of Sector Area and Arc Length Using the Clockwork Context*. *18*(2), 176–186.

Sanita, E., Zulkardi, Z., Susanti, E., & Meryansumayeka, M. (2024). Desain Pembelajaran Refleksi Geometri Menggunakan Pendekatan PMRI dengan Konteks Songket Prada Palembang. *Indiktika : Jurnal Inovasi Pendidikan Matematika*, *7*(1), 246–260. https://doi.org/10.31851/indiktika.v7i1.17196

Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective: The Concept of a Hypothetical Learning Trajectory. Journal for Research in Mathematics Education, 26(2), 114–145. https://www.jstor.org/stable/749205

Sulistyawati, S., & Rofiki, I. (2022). Constructing fractal batik using GeoGebra: An ethnomathematics perspective. *International Journal of Trends in Mathematics Education Research*, 5(2), 97–103.

Yuliardi, R., & Rosjanuardi, R. (2021). Hypothetical learning trajectory in student's spatial abilities to learn geometric transformation. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, *6*(3), 174–190. https://doi.org/10.23917/jramathedu.v6i3.13338

Zulkardi, & Ilma, R. (2006). Mendesain sendiri soal kontekstual matematika. *Prosiding KNM13 Semarang*, 1–7.

Zulkardi, & Ilma, R. I. I. (2018). Developing learning materials with PMRI approach. Journal on Mathematics Education, 9(2), 157–170.