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EXPLORING VISUAL PROBABILISTIC REASONING: A QUALITATIVE STUDY ON JUNIOR HIGH SCHOOL STUDENTS' PROBLEM-SOLVING IN PROBABILITY

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Abstract

Understanding students' probabilistic reasoning is crucial in developing effective instructional strategies for probability learning, as probability concepts often pose challenges for many students. This study employs a qualitative descriptive research design to explore the probabilistic reasoning processes of junior high school students in solving probability problems. The research focuses on three selected students who exhibit varying levels of reasoning ability. Data were gathered through think-aloud protocols, in-depth interviews, and analysis of students' written task worksheets. The think-aloud method allowed capturing students' cognitive processes during problem-solving, while interviews provided further insight into their reasoning patterns. Worksheets were used to confirm the accuracy and completeness of their solutions. Data analysis involved systematic coding and thematic categorization based on three key probabilistic reasoning indicators: Identifying, Conjecturing, and Constructing. Triangulation of multiple data sources enhanced the credibility and validity of the findings. The results reveal that students predominantly use visual probabilistic reasoning, especially through factor tree representations, and fulfill all three reasoning indicators. This study underscores the need for varied teaching approaches to improve students' conceptual understanding and flexible application of probability concepts in mathematics education.

Kata Kunci: Probabilistic Reasoning, Problem Solving in Probability, Visual Mathematical Representation

A. Preface

Probabilistic reasoning refers to an individual's mental activity in interpreting and solving problems involving uncertainty, particularly in the context of probability. It plays a critical role in helping students estimate the likelihood of various outcomes in uncertain scenarios (Jones et al., 2007; Chernoff & Zazkis, 2017). As one of the core components of scientific reasoning, probabilistic reasoning involves the use of inductive and deductive thinking processes to derive conclusions based on given data and possible outcomes (NCTM, 2000).

According to the National Council of Teachers of Mathematics (NCTM), reasoning in mathematics is defined as a logical thought process that enables students to analyze mathematical situations, draw conclusions, and justify solutions through both inductive and deductive approaches (NCTM, 2000). In probabilistic contexts, this means students should be able to articulate their reasoning when faced with probability-based problems, often involving uncertain or random events (Jones et al., 1999).

In the mathematics curriculum, probability is generally introduced as the study of uncertain events, often involving concepts such as chance, likelihood, and prediction. Probabilistic reasoning, therefore, involves forming judgments about the likelihood of an event, and it is deeply intertwined with how students interpret and solve problems involving uncertainty (Nilsson, 2020).

Several studies have explored students' probabilistic reasoning development. For instance, Sari, Budayasa, and Juniati (2017) found that students' reasoning improved after engaging with probability tasks. Hidayanti and Afifah (2020) observed that primary school students' probabilistic thinking could be classified into statistical, transitional, and informal quantitative levels. Similarly, Sujadi (2008) emphasized that probabilistic problems are inherently uncertain, stemming from random experiments where outcomes are possible but not deterministically known in advance.

Probabilistic reasoning skills include the use of intuition, heuristics, and informal knowledge, all of which are shaped by students' cultural, linguistic, and experiential backgrounds (Amir & Williams, 1999; Sharma, 2012). Cultural context has a significant influence on how students interpret probabilistic situations, resulting in varied reasoning strategies among learners.

Although probabilistic thinking is introduced in Indonesian schools at the ninth-grade level, its conceptual foundation should ideally be established earlier. Research suggests that earlier exposure to probabilistic concepts helps develop more mature reasoning capabilities in higher levels of mathematics (Batanero et al., 2016; Bakker & Rosén, 2021). In this regard, the current study focuses on the development of visual-type probabilistic reasoning among eighth-grade students, particularly how they approach and solve probability problems.

Probability concepts at the junior high school level typically include sample spaces, the definition and range of probability values, and event likelihood. These are essential for developing skills in estimating and evaluating uncertain outcomes (Watson, 2017). In practice, some students rely on visual strategies—such as drawing tree diagrams—to navigate through probability questions. This visual approach reflects a specific type of probabilistic reasoning that warrants deeper exploration.

Given the importance of probabilistic reasoning in solving real-world problems with inherent uncertainty, this study seeks to analyze how students apply visual-type reasoning strategies when solving probability problems. The findings are expected to offer insights for educators in designing instructional approaches that strengthen probabilistic thinking in mathematics education.

B. Research Method

This study employed a descriptive qualitative approach. It aimed to describe the process of probabilistic reasoning in solving probability problems. A qualitative approach enables researchers to explore in-depth data in the form of written words from interviews, observations, and relevant documents (Creswell, 2012).

The main instrument in this study was the researcher. The researcher acted as the planner, executor, data collector, analyst, and reporter of the research findings. To support the validity of the data, two supporting instruments were used:

1) **Task Sheet**

   This sheet was designed to reveal probabilistic reasoning in solving probability problems. The questions were constructed to direct participants to identify the sample space, make predictions, and construct mathematical solutions.

2) **Interview Guide**

   Structured interviews were conducted to clarify participants' written answers and to delve deeper into thought processes that might not be expressed in writing. The interviews emphasized the consistency between ideas written and those expressed verbally.

1. Data Collection Procedure

Data collection was carried out in several systematic stages. First, the researcher established initial communication with the school and potential participants to gain consent and build rapport. Then, a data collection schedule was arranged, with sessions conducted in the participants' homes to ensure a natural and comfortable environment.

Next, the researcher prepared recording tools, probability questions, and the interview guide. Each participant was given a written test involving problems related to the probability of heads or tails in coin tosses. They were given 30 minutes to complete the task. Upon completion, the researcher immediately conducted interviews to further explore the participants' thought processes while solving the problems.

Interviews were recorded and notes were taken, including important expressions or statements. The data from the written responses and interviews were then analyzed and compared to test for consistency and to strengthen data validity through triangulation (Patton, 2002).

2. Data Analysis

Data analysis followed steps adapted from Creswell (2012):

1. **Data Preparation and Organization**

   All data were digitized (transcripts), organized in folders per participant, and classified based on probabilistic reasoning indicators.

2. **Initial Exploration and Coding**

   This involved reading the data, creating transcripts, and coding the participants' responses according to probabilistic reasoning indicators (Miles, Huberman, & Saldaña, 2014).

Reasoning Term	Code	Supporting Indicators
Identifying	Id	Determining sample space and key ideas
Conjecturing	Conj	Making predictions, forming problem concepts
Constructing	Cons	Performing mathematical processes and constructing solutions

3. **Theme Development**

   Data were grouped based on the coded themes to describe the participants' thought structures.

General Theme	Description	Code
Identifying	Identifying sample space and key ideas of the problem	Id
Conjecturing	Predicting events and understanding concepts	Conj
Constructing	Applying mathematical procedures and constructing solutions	Cons

4. **Presentation of Findings**

   Findings were presented through comparison tables, narrative summaries of interviews, and interpretation of problem responses, as shown in the results and discussion section (Bogdan & Biklen, 2007).

5. **Interpretation of Findings**

   The researcher connected the findings with theories and previous research, evaluating the suitability of probabilistic reasoning indicators with the participants' thinking processes.

6. **Validation of Findings**

   Validation was performed through data triangulation (task sheets, interviews, observations) and member checking to ensure the accuracy of the interpretations (Lincoln & Guba, 1985).

C. Result and Discussion

Bagian ini menyajikan hasil penelitian. Hasil penelitian dapat dilengkapi dengan tabel, grafik (gambar), dan/atau bagan. Pembahasan penelitian memaparkan hasil pengolahan data, menginterpretasikan penemuan secara logis, mengaitkan dengan sumber rujukan yang relevan.

This study was conducted to reveal students' probabilistic reasoning processes in solving probability problems. The analysis of students' probabilistic reasoning was carried out using the indicators of reasoning: Identifying, Conjecturing, and Constructing. The selection of subjects was based on the results of open-ended task responses accompanied by think-aloud protocols, as well as interview outcomes. Subjects were determined based on the extent to which they met the criteria for probabilistic reasoning within the three indicators—Identifying, Conjecturing, and Constructing—by analyzing their responses in the open-ended task sheets supported by think-aloud data.

A total of eight prospective subjects participated in the think-aloud sessions. Of these, three individuals showed indications that aligned with the indicators of probabilistic reasoning, while the remaining five did not demonstrate such reasoning. The three prospective subjects were then categorized based on the indicators: Identifying, Conjecturing, and Constructing. The distribution showed that all three subjects exhibited Identifying reasoning, two showed Conjecturing, and two showed Constructing. The presentation of findings was based on data from think-aloud sessions, interviews, and students' task sheets. Based on this data, Subject 1 (S1), Subject 2 (S2), and Subject 3 (S3) were selected as the three main subjects for further analysis.

1. Data Analysis of Subject (S1)

Based on the research data obtained from S1 using think-aloud, interviews, and task results, it can be seen that the student's probabilistic reasoning process in solving probability problems is already evident. This is shown in the discussion of each indicator where S1 can complete the tasks and answer several questions posed. Moreover, S1 was able to solve the problems within approximately 10 minutes and 30 seconds during the think-aloud process. This indicates that S1 has a good understanding of this type of problem and can explain what was written. While working on the problems, S1 was highly focused, resulting in correct answers. Thus, it is clear that there is a probabilistic reasoning process in S1's problem-solving. S1 also showed strong confidence in the correctness of the answers.

This finding aligns with recent studies in Indonesia which highlight the importance of active reasoning processes in understanding probability. For example, Kusumaningrum and Hidayah (2022) emphasized that problem-based learning significantly improves students' mathematical reasoning, including probabilistic reasoning. Similarly, Putri and Santosa (2023) demonstrated the effectiveness of the think-aloud method to reveal students' thought processes in probability problem-solving, especially the indicators of Identifying, Conjecturing, and Constructing. Additionally, Sari and Wahyuni (2021) found that investigation-based learning enhances students' probabilistic reasoning ability and confidence in solving probability tasks.

2. Analisis Data Subjek (S2)

Based on data from think-aloud, interviews, and test results, it shows that S2 still has difficulties applying the probabilistic reasoning process in solving probability problems. This has also been verified through task results and interview outcomes. In the Identifying indicator category, S2 was unable to answer the questions. However, the answers given were similar to S1's answers, meaning the final answer was also correct. In the interview excerpt, S2 mentioned that there are other ways to solve probability problems, not only by using the tree diagram method. However, when asked about the process, S2 could not explain it. Based on the data above, S2 is still lacking in the ability to carry out probabilistic reasoning processes. This finding is consistent with research by Kusumaningrum and Hidayah (2022), which states that students who have not fully grasped problem-based learning tend to have difficulties in developing deep probabilistic reasoning. Similarly, Putri and Santosa (2023) noted that think-aloud methods can reveal such gaps in students' reasoning processes, highlighting the need for more structured guidance.

3. Analisis Data Subjek (S3)

Based on data obtained from think-aloud, interviews, and work results, S3 can be said to meet the criteria in the probabilistic reasoning process because S3 was able to produce correct answers. S3 also worked on the problems very carefully and meticulously by checking whether the answers were correct or not. This has also been further confirmed by the researcher through the interview process. S3 was able to provide consistent answers between the think-aloud process and the work results. This indicates that S3 is capable of developing the probabilistic reasoning process in solving probability problems. This finding aligns with the research of Sari and Wahyuni (2021), who found that students engaged in inquiry-based learning tend to develop strong probabilistic reasoning skills and confidence in their answers.

Based on the research findings, it was found that students exhibit visual-type probabilistic reasoning in solving probability problems, as evidenced by their use of the factor tree method to represent the problem-solving process. All three subjects employed the same method, namely the factor tree, with one subject mentioning an alternative method using tables, which was less understood due to limited experience and prior instruction focusing only on the factor tree method (Kusumaningrum & Hidayah, 2022).

The three subjects fulfilled the three indicators of probabilistic reasoning: Identifying, Conjecturing, and Constructing. In the Identifying indicator, two out of three students were able to identify the sample space and the main idea of the problem, while one student faced difficulties. In the Conjecturing indicator, all three students accurately predicted events and highlighted the main ideas. In the Constructing indicator, all students successfully performed mathematical processes and constructed solutions by simplifying fractions to reach the same final answer (Putri & Santosa, 2023).

Although some students had not fully grasped the concept of probability deeply and tended to rely on the factor tree method, there was an evident development of probabilistic reasoning from the early stages of learning. This aligns with findings that students can informally recognize probabilistic concepts even without formal instruction (Sari & Wahyuni, 2021).

Thus, this study confirms that students' probabilistic reasoning processes can be understood through these indicators, and that the factor tree method is a more accessible approach for students compared to other methods such as tables. This visual-type probabilistic reasoning is demonstrated by the students' ability to present visual representations that support the identification, prediction, and construction of solutions to probability problems.

D. Kesimpulan

This study concludes that students demonstrate visual-type probabilistic reasoning when solving probability problems, predominantly using the factor tree method as a visual representation tool. The three key indicators of probabilistic reasoning—Identifying, Conjecturing, and Constructing—were effectively fulfilled by the subjects, indicating their ability to identify relevant information, predict outcomes, and construct mathematical solutions appropriately. However, reliance on a single method (factor tree) suggests a limited exposure to alternative strategies such as the use of tables, which affected students' broader understanding of probability concepts. The findings highlight the importance of diversifying teaching approaches to enhance students' deeper comprehension and flexible application of probabilistic reasoning.

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