

UNDERSTANDING MORE MATHEMATICS PEDAGOGICAL CONTENT KNOWLEDGE

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Abstract

The effectiveness of mathematics learning activities in the classroom is influenced by many factors such as the psychology of student learning, the level of material being taught, the classroom atmosphere, and the knowledge of the mathematics teacher. A mathematics teacher must at least have in-depth knowledge of mathematics content, strong pedagogical knowledge, and sophisticated technology knowledge. The integration of mathematics content knowledge and pedagogic knowledge makes mathematics pedagogical content knowledge (MPCK) unique as teachers must understand science, philosophy, and dimensions of mathematics, and then integrate it into learning activities so that learning objectives are achieved. A systematic review approach was carried out in this study by reviewing 29 out of 221 articles based on the results of analysis from searches with various keywords. The results showed that MPCK has specifications in terms of definition, aspects, linkages with various aspects of learning, methods of assessment and evaluation, improving method, teacher characteristics, and the impact on a mathematics lesson. This research supports mathematics teacher's efforts to implement effective mathematics learning by understanding more MPCK and optimizing MPCK.

Keyword: teacher competence, mathematics pedagogical content knowledge, MPCK, systematic review

A. Introduction

The development of mathematics learning goes in harmony with the demands of the ages which are growing along with human needs. In the era of post-modernism, learning mathematics is seen as a constructive process in which learning involves social and cultural experiences from learning subjects, namely educators and students to find a concept (Ernest, 2004). The philosophy of learning mathematics in a constructivist way is the result of a development that is adapted to the era after it was mostly done with behaviourism prior (Lerman, 1989). The institution not only demanded for cognitive domain achievements, but

also affective skills such as creativity, collaboration, critical thinking, communication, and social skills as the results of mathematics learning (Triana dkk., 2020; Zakiah & Fajriadi, 2020).

Mathematics teachers often have difficulty planning constructivist learning, especially in terms of looking for contextual problems that can be related to mathematics lessons according to students' experiences as part of the knowledge construction process through mathematical reasoning and connections (Christou dkk., 2004). Various efforts have been developed to accommodate the construction process of mathematics, such as strategies for packaging materials, delivering materials, managing classes, and evaluating them. Despite found many difficulties, a teacher is given provisions in the form of teacher competencies that must be fulfilled during education prior to starts teaching.

Teacher competencies are the embodiment of knowledge to organize teaching and learning effectively which consist of knowledge of content, pedagogic, organization, psychological, and counselling (Baumert & Kunter, 2013). A teacher has been provided with knowledge about the material to be taught, knowledge about various material management strategies, management of learning subjects, class management, delivery of material, as well as knowledge of student learning psychology which consists of the affective and psychomotor domains. Mathematics teacher competence affects the broaden context of material to be taught, learning effectiveness, students' learning achievement, as well as professional development (Ariyanti, 2015).

In practice so far, mathematical content knowledge and pedagogical knowledge are often separated. Mathematical content knowledge or professional knowledge is interpreted as the ability to fluent mathematics concepts, principles, and its relation (Indonesia Consitution No. 14, 2005). Mathematical content knowledge can also be interpreted as the number of mathematical knowledge organizations owned by teachers including conceptual knowledge, procedural knowledge, and the relationship of each concept in it (Shulman, 1986). Meanwhile, pedagogical content knowledge is stated as the ability to understand how to organize and represent a topic, material, or problem in learning that is able

to accommodate differences in students both in terms of abilities and interests (Shulman, 1987).

Pedagogical content knowledge is not only knowledge about how to teach and about what is taught in schools for students but also knowledge about students both cognitive and affective aspects, as well as knowledge of the social, cultural and political conditions of the students' learning environment (Cochran dkk., 1993). Pedagogical content knowledge can make learning effective as this knowledge includes how to deliver learning material, interact with students, as well as reflect on the instructions during learning (Walshaw, 2012).

The combination of mathematics content knowledge and pedagogical content knowledge makes mathematics pedagogical content knowledge (MPCK) unique because in addition to having to understand science, philosophy, and dimensions of mathematics, they also have to be able to integrate them into learning activities so that learning objectives are achieved (Astuti & Jailani, 2020; Novikasari, 2020). In order to describe this combination, it is necessary to study the definition of MPCK, MPCK aspects, its relation to other aspects such as affective, how to do an assessment, how to improve MPCK, the characteristics of teachers with good MPCK and the impact on learning.

B. Method

This research is a systematic review with a qualitative approach. The subjects of the research were journal articles and proceedings on MPCK such as definitions, aspects, assessments, ways to improve, relations with other aspects of learning, characteristics of teachers with good MPCK, and the impact on learning collected from various international journals. The steps taken in the research are finding initial sources according to the topic, looking for further sources that are more detailed regarding the objectives, reading the articles that have been obtained, and conducting a synthesis from a literature review (Gall dkk., 2003).

The search for the article used databases involving Google Scholar, ERIC, Springer, Sagepub, Dergipark, as well as Tandfonline to simplify the search process with the various features provided. The process has started in October 2022 until December 2022 which limited to articles published from 2007 and

using English. Some of the keywords used to search articles are MPCK, MPCK in teaching mathematics, mathematics pedagogical content knowledge (MPCK), elements of mathematics pedagogical content knowledge, mathematics pedagogical content knowledge aspects, and mathematics pedagogical content knowledge evaluation. In the process of searching for articles, the data is exported into a Microsoft Excel file to make a complete recapitulation, then a duplicate article is checked with the remove duplicate feature in the software so that the data is not duplicated.

C. Result and Discussion

The results of article search were 221 articles related to MPCK and then selected according to the research objectives. A total of 29 articles that match the research objectives of the review process were successfully attained (Table1). From these various articles, a discussion of MPCK was compiled in the category of MPCK definition, MPCK aspects, linkages with other aspects such as affective, assessment and evaluation methods, how to improve teacher’s MPCK, as well as the characteristics of teachers with good MPCK and the impact on learning outcomes.

Tabel 1. Reviewed Articles on Every Aspect

Author	Definition	Aspect	Other aspect related	To Assess	To Improve	Teacher Character	Impact on learning
Aksu & Kul (2016)	√			√	√		
Alfaro & Joutsenlahti (2020)	√						
Baki & Arslan (2017)	√	√		√	√		√
Blomeke (2011)							√
Blomeke, et al. (2014)	√	√	√		√		√
Buchholtz & Kaiser (2013)	√	√			√		
Buchholtz (2017)				√	√		
Döhrmann, et al. (2012)		√		√		√	
Ekawati, et al. (2022)		√				√	
Ekawati, et al. (2018)	√	√		√		√	√
Gagsteiger, et al. (2019)	√			√			
Hsieh (2013)		√					
Hoth, et al. (2017)		√	√				√
Kaarstein (2014)		√		√			
Kaiser, et al. (2016)			√				
Kow (2018)	√	√		√			
Kwong, et al. (2007)				√	√		
Laschke (2013)			√				
Lin & Lee (2016)		√			√		√
Luna & Aclan (2015)		√	√	√	√		√

Author	Definition Aspect	Other aspect related	To Assess	To Improve	Teacher Character	Impact on learning
Moh'd, et al. (2021)						√
Msimango, et al. (2020)	√				√	
Norton (2017)		√				
Novikasari (2020)	√				√	
Qian & Youngs (2015)					√	
Senk, et al. (2012)	√					
Shahbari (2017)			√			
Sintema & Marban (2020)	√					
Yang, et al. (2021)		√				√

Definition of MPCK

MPCK has ample different definitions based on a combination of MCK and PCK as well as a synthesis of MCK specifically for education in schools. MPCK is defined as all the abilities that teachers have in learning and teaching mathematics (Alfaro & Joutsenlahti, 2020). Another notion about MPCK is didactic mathematics, namely mathematics in the field of education (Buchholtz & Kaiser, 2013). This means that the content of mathematics is limited to the level of education, age, and student psychology. Therefore, mathematics teachers need to have MPCK skills, namely the ability to characterize students, organize learning, and provide instructions in learning (Baki & Arslan, 2017).

Ekawati et al. (2018) described that MPCK is a combination and complex interaction between mathematics and pedagogical content to build an understanding of how certain content is structured and presented to students. A teacher must master the concepts of mathematics in education, their relationship with other concepts, as well as various ways of presenting them according to students. Aksu & Kul (2016) explained MPCK as the teacher's ability to understand content to identify learning activities and difficulties experienced by students and then create strategies to improve.

In learning with a large group system there are plenty of different student characteristics in learning so we need a way for teachers to be able to identify as an effort to make learning effective. This is in accordance with the opinion of Blömeke et al. (2014) that teachers must have MPCK, namely knowledge about how to present mathematics to students, which is very likely some have problems

understanding difficulties. More operationally Kow (2008) argues that MPCK is the ability to teach mathematics so that students can learn effectively. This can be executed by creating appropriate learning situations for students, analyzing students' mathematical development, and providing spontaneous support in learning (Gasteiger dkk., 2019).

Aspects of MPCK

As a combination of MCK and PCK, MPCK has aspects that are slices or a combination of the two. The MPCK aspects according to Lin & Lee (2016) composed of mathematics content knowledge, knowledge of students' cognitive abilities towards mathematics, knowledge of giving mathematics learning instructions, knowledge of making mathematics assessment instruments, and knowledge of teacher responsibilities. MPCK aspects also include the ability to understand mathematical topics in their structure and interrelationships; the ability to build several mathematical representations, methods, explanatory procedures; and finally, the ability to anticipate student misconceptions through careful learning planning (Novikasari, 2020).

The MPCK aspects consist of the ability to teach mathematics to someone, the ability to represent mathematical ideas, the ability to explain mathematical ideas, and the ability to identify student understanding (Hsieh, 2013). Luna & Aclan (2015) emphasized that aspects of MPCK include the ability to plan lessons, the ability to organize learning situations in class, and the ability to facilitate learning. Broadly speaking, the aspects in MPCK include knowledge about mathematics learning carried out by students along with knowledge in creating a good mathematics learning environment (Msimango dkk., 2020).

In MPCK there are also aspects of the ability to make assignments according to students' abilities, the ability to choose learning strategies according to needs, the ability to understand difficulties and misconceptions experienced by students, as well as the ability to provide guidance to students who experience difficulties (Buchholtz & Kaiser, 2013; Ekawati dkk., 2022). The ability to make assignments/projects that make students active flexibly according to needs and situations is also an aspect of MPCK (Ekawati dkk., 2018). In addition, the

MPCK aspect also includes the ability to evaluate and interpret evaluation results for students and provide feedback (Baki & Arslan, 2017).

Blömeke et al. (2014) along with Sintema & Marbán (2020) provide views on aspects of MPCK sequentially consisting of the ability to identify students' initial conditions, choose certain content and concepts related to learning, apply mathematical language to convey mathematical ideas, utilize various learning strategies, identify student misconceptions, and provide feedback. Other ideas regarding different aspects of MPCK are proposed by Senk et al. (2012) which contain knowledge of mathematics according to the educational curriculum, knowledge of making plans for mathematics learning activities, knowledge of using mathematics in learning.

MPCK aspects also consist of the ability to apply learning processes and assessments in accordance with the curriculum, to design sequential learning activities, as well as to choose activities that are suitable for the material and students (Kow, 2008). The suitability of these aspects with the curriculum is due to the influence of the curriculum on teachers in planning learning with certain learning strategies (Kaarstein, 2014). The link between MPCK and the curriculum was also presented by Döhrmann et al. (2014) that MPCK consists of aspects of mathematical knowledge in the curriculum, the ability to identify important points in learning programs and their interrelationships in the curriculum, the ability to choose learning approaches and methods that are appropriate to the curriculum, and the ability to analyze and evaluate student tasks.

Hoth et al. (2017) added that what is included in the MPCK aspect is the ability to identify mathematical ideas in learning and make learning sequences, also make assignments that challenge students to be creative. The combination of MPCK aspects from the various opinions above can be combined in the ability to understand in-depth material such as understanding concepts and their interrelationships, representing, and conveying mathematical ideas in the language of mathematics; the ability to organize material using various methods according to student character so that learning is effective; carry out assessments and evaluations both in the learning process and in learning outcomes.

Linkages with various aspects of learning

MPCK has a strong relationship with self-regulated learning and teacher motivation (Kaiser dkk., 2016). Teachers with good MPCK are capable to independently learn content along with innovative learning strategies to improve teaching skills and motivate themselves to always be able to teach well. In addition to learning motivation and independence, MPCK also has a link to the creativity and self-confidence of mathematics teachers which can be formed through experience in teaching (Blömeke dkk., 2014; Hoth dkk., 2017; Yang dkk., 2021).

Convinced with the abilities possessed, teachers can be more confident in teaching and make various kinds of assignments according to students' ability levels and provide appropriate feedback in multifarious ways. Belief in one's ability to complete a job is one aspect of self-efficacy (Norton, 2017). Confidence in self-ability can spur someone to do their best in their job (Bandura, 1977). Moreover, MPCK also has a relationship with the habits and behavior of a teacher (Luna & Aclan, 2015). Teachers become more enthusiastic and ready for a change that may arise due to a change in curriculum or policy.

Other aspects related to MPCK are socio-cultural conditions and gender (Laschke, 2013). As it is known that each region has different social and cultural characteristics, there are those who fully support the implementation of education, there are also those who are less enthusiastic about education. This certainly has an effect on teachers or prospective teachers in an effort to organize learning activities and learn to develop MPCK abilities continuously. Regarding gender, from the results of a survey performed by Laschke (2013), female teachers or prospective teachers tend to have better MPCK skills than male teachers.

Assessment and evaluation

To find out the MPCK level of teachers or prospective teachers, it is necessary to carry out an assessment and evaluation to provide advice to teachers or prospective teachers. There are various forms of MPCK assessment, both in writing and in practice through observation. The form of written MPCK assessment can be with multiple choice instruments, complex multiple choice, or structured essay questions from various indicators or aspects of MPCK

(Döhrmann dkk., 2012; Kaarstein, 2014). An open-ended form of the test can also be administered to assess the MPCK of teachers or prospective teachers (Aksu & Kul, 2016; Luna & Aclan, 2015).

The application of essay questions to measure MPCK makes it possible to explore the depth of knowledge possessed by teachers or prospective teachers of mathematics by referring to aspects of understanding the structure and relationships of mathematics, understanding students' difficulties and misconceptions, student solution strategies and appropriate responses, knowledge of alternative teaching methods and representations various explanatory concepts of numbers, arithmetic operations, geometry, and logic problems (Shahbari, 2017). An essay questions using problems in teaching learning mathematics used by Kwong et al. (2007) to measure MPCK with indicator teacher's own understanding of mathematical structure and connections, teacher's knowledge of a range of alternative representations of concepts for purpose of explanation, teacher's ability to analyze the cognitive demands of mathematical tasks on learners, and teacher's ability to understand and take appropriate action for children's learning difficulties and misconceptions. Furthermore, the form of a questionnaire test can also be carried out as a form of teacher self-assessment regarding aspects of MPCK (Buchholtz, 2017).

In addition to written tests, to assess MPCK face-to-face interviews can be deployed to find out aspects of MPCK that are mastered theoretically by teachers and able to prevent factors such as guessing choice (Baki & Arslan, 2017; Gasteiger dkk., 2019). Hereafter, to see from the perspective of the teacher's practical ability in teaching, MPCK assessment can be done by observing both directly and indirectly using video recordings of learning so that teaching and learning activities can be more natural (Baki & Arslan, 2017; Ekawati dkk., 2018; Kow, 2008).

How to improve MPCK

MPCK ability cannot be obtained in an instant but requires a continuous process step by step. One of the steps to improve the MPCK ability of prospective mathematics teachers is to incorporate MPCK aspects into the learning curriculum at lectures (Luna & Aclan, 2015). Aspects of MPCK need to be introduced to

prospective mathematics teachers too (Lin & Lee, 2016). Things that need to be considered in lectures are the level of depth of mathematics material, depth of mathematics learning methods, and opportunities to practice learning outcomes (Qian & Youngs, 2015). The deepening of mathematics material needs to be done in anticipation of changes in the content of learning materials due to changes in curriculum or policies (Blömeke dkk., 2014).

Methods to improve the MPCK of future mathematics teacher students in lectures can be carried out by integrating the problem based learning (PBL) method so that learning becomes meaningful considering that aspects of MPCK often become problems in learning activities (Novikasari, 2020). In the process of teaching practice by students by applying MPCK it is necessary to provide feedback as a process of reflection and learning (Baki & Arslan, 2017). The MPCK capacity building program can also be carried out with a mentoring program from teachers who are experienced and have good competence and focus on process not results (Msimango dkk., 2020).

Other ways to strengthen prospective teachers' mathematical content knowledge can be carried out by providing special programs for MPCK development such as courses for learning mathematics in a new way or a better understanding of mathematics (Buchholtz, 2017; Buchholtz & Kaiser, 2013; Kwong dkk., 2007). In line with this, a training outside of lecture time is needed to hone MPCK skills from both student teacher candidates and math teachers (Aksu & Kul, 2016). The assorted efforts made to improve the teacher's MPCK ability are expected to be able to influence learning both from the students' and the teacher's side.

Teacher characteristic with good MPCK and impact on learning

Teachers with good MPCK are certainly different from teachers with poor MPCK. One of the characteristics of teachers who have good MPCK is the ability to observe and analyze students' initial abilities, affective aspects of students in learning, and psychomotor aspects (Döhrmann dkk., 2012). Regarding learning materials, teachers who have good MPCK are able to make various kinds of representations of the concepts being taught, connect one concept with other

concepts from different branches of knowledge, also identify conceptual errors experienced by students in learning (Ekawati dkk., 2018).

Teachers with good MPCK are able to design learning strategies according to the material, provide a variety of different assignments that can even serve as feedback, and are able to identify students' level of understanding in learning (Ekawati dkk., 2022). Good teachers MPCK certainly have an impact on the quality of learning conducted. One of the effects of a good teacher's MPCK is to make learning effective so as to increase student achievement (Blömeke dkk., 2011; Lin & Lee, 2016; Luna & Aclan, 2015; Yang dkk., 2020). Teachers with better MPCK are able to be more effective in organizing mathematics learning (Moh'd dkk., 2021).

The ability of a good teacher's MPCK in providing appropriate reactions to students such as the ability to provide appropriate feedback to students is able to influence student learning activities, drive students courage to make opinions and presentations, as well as motivate students (Baki & Arslan, 2017; Hoth dkk., 2017). It can make students enjoy and not afraid learning mathematics. Last, the teacher's ability to manage class and learning well can increase the creativity and independence of student learning (Blömeke dkk., 2014; Hoth dkk., 2017).

D. Conclusion

MPCK can be concluded as a combination of mathematics content knowledge material in learning along with knowledge in organizing and conveying material to students with distinct characteristics effectively. Aspects of MPCK are not only limited to the ability to understand in-depth material such as understanding concepts and their interrelationships, representing and conveying mathematical ideas in the language of mathematics but also the ability to organize material using various methods according to student characteristics so that learning is effective as well as conducting assessments and evaluations both in the process learning and on learning outcomes. MPCK is related to other learning aspects such as affective and psychomotor. These aspects include learning independence, motivation, self-confidence, self-efficacy, as well as creativity.

To find out the MPCK level of each teacher, assessment and evaluation can be carried out using various assessment techniques such as tests of the ability of teacher knowledge through objective instruments and essays with open-ended questions, questionnaires, interviews, and observations of the implementation of learning conducted by the teacher. In order for teachers and prospective mathematics teachers to have good MPCK, various efforts can be made both at the educational level and after becoming a teacher. Several things that can be done to improve the MPCK of teachers and prospective mathematics teachers are by introducing MPCK from the time of education through integration into the educational curriculum, providing opportunities for prospective teachers to practice more MPCK, establishing a mentoring program involving experienced and competent teachers, providing feedback in practice, as well as providing regular training to mathematics teachers.

The characteristics of teachers who have good MPCK include being able to observe and analyze students' initial abilities, affective aspects of students in learning, as well as psychomotor aspects, being able to make various kinds of representations of the concepts being taught, linking one concept to another in a different branch of science, as well as identify conceptual errors experienced by students in learning. The good impact of MPCK from mathematics teachers is that they are able to influence student learning activities, encourage students to make opinions and presentations, motivate students, increase creativity and independence of student learning, as well as make learning effective so as to increase student learning achievement.

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